Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Find the value of k that satisfies the following equation:

$$\det\begin{pmatrix} 2a_1 & 2a_2 & 2a_3 \\ 3b_1+5c_1 & 3b_2+5c_2 & 3b_3+5c_3 \\ 7c_1 & 7c_2 & 7c_3 \end{pmatrix} = k\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}.$$

$$\| $$

$$7\cdot 2 \det\begin{pmatrix} a_1 & a_2 & a_3 \\ 3b_1+5c_1 & 3b_2+5c_2 & 3b_3+5c_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$\| $$

$$7\cdot 2\cdot \det\begin{pmatrix} a_1 & a_2 & a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$3\cdot 7\cdot 2\cdot \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \Rightarrow k = 42.$$

2. Prove, referring to properties of the determinant, that for $A \in M^{n\times n}(\mathbb{F})$, $\det(-A) = \det(A)$ if n is even, and $\det(-A) = -\det(A)$ if n is odd.

$$\begin{vmatrix} -a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & -a_{22} & \cdots & -a_{2n} \\ & & & \\ -a_{n1} & -a_{n2} & \cdots & -a_{nn} \end{vmatrix} = - \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ -a_{21} & -a_{22} & \cdots & -a_{2n} \\ \vdots & & & \\ -a_{n1} & -a_{n2} & \cdots & -a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ -a_{n1} & -a_{n2} & \cdots & -a_{nn} \end{vmatrix} = \cdots \underset{\text{(induction on } n)}{=} (-1)^n \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

(used the scalar multiple part of the multi-linear property.)

Please turn over...

3. True or false (you do not need to show your work):

(a) The function $\det : M^{n\times n}(\mathbb{F}) \to \mathbb{F}$ is a linear transformation

(b) If B is a matrix obtained from a square matrix A by interchanging any two rows, then $\det(B) = -\det(A)$.

(c) If B is a matrix obtained from a square matrix A by multiplying a row of A by a scalar, then $\det(B) = \det(A)$.

(d) If B is a matrix obtained from a square matrix A by adding k times row i to row j, then $\det(B) = k\det(A)$.

Answers:

(a) F (multi-linear ≠ linear)

(b) T

(c) F (det is multiplied by a scalar.)

(d) F (the det. does not change.)