Thu, Sept 11, 2014 **MATH 462, Quiz 2** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Let $S = \{u_1, u_2, \ldots, u_n\}$ be a finite set of vectors. Prove that S is linearly dependent if and only if $u_1 = 0$ or $u_{k+1} \in \operatorname{span}\{u_1, u_2, \ldots, u_k\}$ for some $k \in \{1, \ldots, n\}$.

"$\Rightarrow$" Assume S is lin. dependent

Then $\exists\ a_1 \ldots a_n$, not all zero:

$$a_1 u_1 + \ldots + a_n u_n = 0$$

Let $k = \max\{i : a_i \neq 0\}$.

If $k = 1$ then

$$a_1 u_1 = 0,\ a_1 \neq 0 \Rightarrow u_1 = 0$$

If $k > 1$ then

$$a_1 u_1 + \ldots + a_k u_k = 0$$

$$\Rightarrow u_k = -\frac{a_1}{a_k} u_1 - \ldots - \frac{a_{k-1}}{a_k} u_{k-1}$$

$$\Rightarrow u_k \in \operatorname{span}\{u_1 \ldots u_{k-1}\}.$$

"$\Leftarrow$" $u_1 = 0 \Rightarrow$ S is linearly dependent

$$(1 \cdot u_1 + 0 \cdot u_2 + \ldots + 0\, u_n = 0)$$

$u_{k+1} \in \operatorname{span}\{u_1 \ldots u_k\} \Rightarrow$

$$a_1 u_1 + \ldots + a_k u_k = u_{k+1}$$

$$a_1 u_1 + \ldots + a_k u_k - u_{k+1} = 0$$

$\Rightarrow$ S is linearly dependent.

Please turn over...

2. Give an example of three linearly dependent vectors in $\mathbb{R}^3$ such that none of the three is a multiple of another. Justify your answer.

$v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (1,1,0)$

Then $\quad v_1 + v_2 - v_3 = 0$ — nontrivial linear relation

However
$$v_1 \neq c v_2$$
$$v_1 \neq c v_3$$
$$v_2 \neq c v_1$$
$$v_3 \neq c v_1$$
$$v_2 \neq c v_3$$
$$v_3 \neq c v_2$$

3. The set of solutions to the system of linear equations

$$x_1 - 2x_2 + x_3 = 0$$
$$2x_1 - 3x_2 + x_3 = 0$$

is a subspace in $\mathbb{R}^3$. Find a basis for this subspace.

$$\begin{array}{c} \times(-2) \\ + \end{array} \begin{pmatrix} 1 & -2 & 1 \\ 2 & -3 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{array}{c} + \\ \times 2 \end{array}$$

$$\to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} \quad \text{(RREF)}$$

Then $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} t$ $\quad$ (t - parameter)

Basis: $\{(1,1,1)\}$ $\quad$ (one-dimensional subspace)