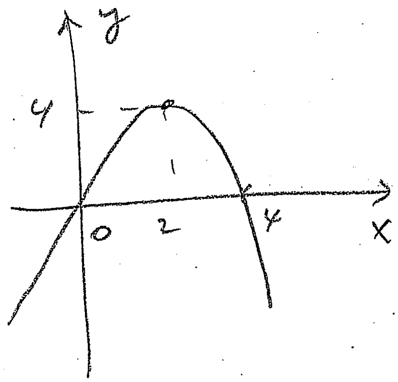
Thu, Oct 31, 2019 **MATH 450A** **Quiz 7**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Find the intervals I_1, I_2 on which f is either increasing or decreasing, and the corresponding intervals J_1, J_2 on which the inverse functions g_1, g_2 are defined. Find the expressions for g_1, g_2 and sketch their graphs.



$f(x) = 4x - x^2.$

$f'(x) = 4 - 2x = 2(2-x)$

$f'(x) > 0 \iff x < 2$

$f'(x) > 0 \iff x \le 2$

$I_1 = (-\infty, 2]$ (f incr.)

$I_2 = [2, \infty)$ (f decr.)

$f(x) = y \iff 4x - x^2 = y$

$\iff x^2 - 4x + y = 0$

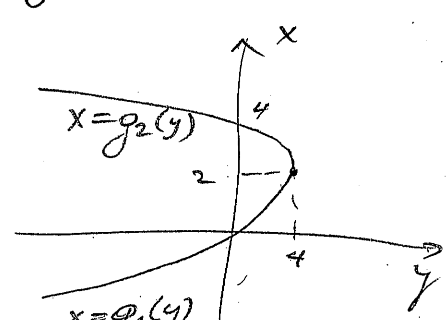
$\iff (x-2)^2 = 4 - y$

$\iff x = 2 \pm \sqrt{4-y}$

$J_1 = J_2 = (-\infty, 4]$

$g_1(y) = 2 - \sqrt{4-y}$

$g_2(y) = 2 + \sqrt{4-y}$



Please turn over...

2. Is the function f differentiable at $x = 0$: $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$?

If 'yes' find $f'(0)$, if 'no' give a proof.

$$f'(0) = \lim_{h \to 0} \frac{h^2 \sin \frac{1}{h} - 0}{h}$$

$$= \lim_{h \to 0} h \sin \frac{1}{h} = 0$$

Since $\left| h \sin \frac{1}{h} \right| \leq |h| \xrightarrow[h \to 0]{} 0$.

Therefore $f'(0)$ exists and is $= 0$.
(f is differentiable at $x=0$)

3. Suppose $f : x \mapsto (x+1)^3$ and $x_0 = 0$ in the Fundamental Lemma of Differentiation. Show that $\eta(h) = 3h + h^2$.

$$f(x_0+h) - f(x_0) = f'(x_0)h + \eta(h)h$$

$$(h+1)^3 - 1 = 3h + \eta(h)h$$

$$f'(x) = 3(x+1)^2; \quad f'(0) = 3$$

$$\eta(h) = \frac{(h+1)^3 - 1 - 3h}{h} \quad (h \neq 0)$$

$$= \frac{h^3 + 3h^2 + \cancel{3h} + \cancel{1} - \cancel{1} - \cancel{3h}}{h} \quad (h \neq 0)$$

$$= h^2 + 3h \quad (h \neq 0)$$

Since $\eta(0) = 0$,

$$\eta(h) = 3h + h^2.$$