Thu, Sept 10, 2019 **MATH 450A** **Quiz 2**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

Symbols a, b, c, d, x are used to refer to arbitrary elements of an ordered field F.

1. If $a \neq 0$ prove that $a^{-1} \neq 0$, and furthermore $(a^{-1})^{-1} = a$.

By def. of a^{-1}, $(a^{-1})a = 1 \neq 0$

Since $0 \cdot a = 0$, $a^{-1} = 0$ is impossible.

$\Rightarrow a^{-1} \neq 0$.

Since $a^{-1}x = 1$ has a unique solution

$x = a$ (multiply $a^{-1}x = 1$ by a)

and $a^{-1}(a^{-1})^{-1} = 1$, we must have

$(a^{-1})^{-1} = a$.

2. If $a > b$ and $c > d$ prove that $a + c > b + d$.

$a > b \Rightarrow a + c > b + c$

$c > d \Rightarrow b + c > b + d$ (Thm 1.16)

By transitivity of inequality "$>$"

$a + c > b + c$, $b + c > b + d \Rightarrow a + c > b + d$.

Please turn over...

3. Solve:

$$\left|\frac{3-2x}{2+x}\right| < 2.$$

$$\iff \quad -2 < \frac{3-2x}{2+x} < 2$$

$$\iff \left[\begin{array}{ll} -2(2+x) < 3-2x < 2(2+x), & 2+x>0 \\ -2(2+x) > 3-2x > 2(2+x), & 2+x<0 \end{array}\right.$$

$$\iff \left[\begin{array}{llll} -4 < 3, & 4x > -1, & x > -2 \\ -4 > 3, & 3-2x > 2(2+x), & 2+x<0 \end{array}\right.$$

Second case is never true; first case

$$\iff \quad x > -\frac{1}{4}$$

4. Prove that for any a and b,

$$|a| - |b| \leq |a-b|.$$

$$|x+y| \leq |x| + |y|$$

let $x = a-b$, $y = b$, $x+y = a$

$$\Rightarrow \quad |a| \leq |a-b| + |b|$$

$$\Rightarrow \quad |a| - |b| \leq |a-b|$$

$$\Rightarrow \quad |a-b| \geq |a| - |b|.$$