Tue, Sep 3, 2019 **MATH 450A** **Quiz 1**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Use truth tables to show that for any logical statements P and Q

$$(P \Rightarrow Q) \equiv (\overline{Q} \Rightarrow \overline{P}).$$

P	Q	$\overline{Q}$	$\overline{P}$	$P \Rightarrow Q$	$\overline{Q} \Rightarrow \overline{P}$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

table of outcomes match

2. Prove that for any sets A, B, C $A \cup B = A$ if and only if $B \subseteq A$.

1) $A \cup B = A$, WTS $B \subseteq A$

$\forall x \quad x \in B \Rightarrow x \in B$ or $x \in A \Rightarrow x \in A \cup B$

$\Rightarrow x \in A$

2) $B \subseteq A$, WTS $A \cup B = A$

$\forall x \quad x \in A \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B$

$\forall x \quad x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ or $x \in A \Rightarrow x \in A$.

Please turn over...

3. Let f be defined by $f(x) = (x+1)^2/x$ for all $x \neq 0$. Find $f((-1,0))$ and $f^{-1}([1,3])$. [Graphical solution is OK.]

$$\frac{(x+1)^2}{x} = y$$

$$x^2+2x+1 = yx$$

$$x^2+(2-y)x+1=0$$

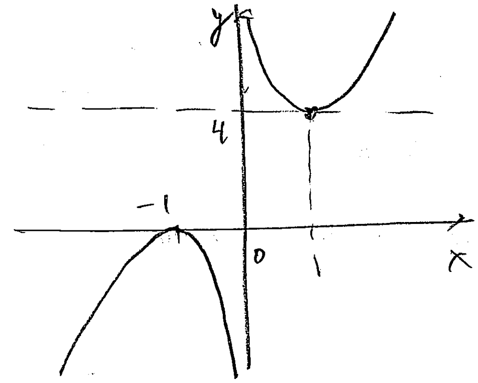
$$(2-y)^2-4 \geq 0$$

$$(2-y)^2 \geq 4$$

$$|2-y| \geq 2$$

$$y \geq 4 \text{ or } y \leq 0$$

$$\Rightarrow f^{-1}([1,3]) = \emptyset$$



$$f((-1,0)) = (-\infty, 0)$$

4. Prove that if $f : X \to Y$ is a one-to-one function and $A_1, A_2 \subseteq X$ then

$$f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2).$$

$\forall y \quad y \in f(A_1) \cap f(A_2) \Rightarrow y \in f(A_1)$ and $y \in f(A_2)$.

$\Rightarrow \exists x_1 \in A_1,\ y = f(x_1),\ \exists x_2 \in A_2,\ y = f(x_2)$

Since f is 1-to-1, $y = f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2 \quad \Rightarrow x_1 \in A_1 \cap A_2$.

$\Rightarrow y = f(x_1) \in f(A_1 \cap A_2)$.