Extra Credit: Problems on Uniform Continuity

1. True or false: If $f : [0, \infty) \to \mathbb{R}$ is continuous on its domain and

$$\lim_{x \to +\infty} f(x) = 0$$

then f is uniformly continuous on $[0, \infty)$. If true give a proof; if false provide a counterexample.

2. Suppose that $f : [0, \infty) \to \mathbb{R}$ is uniformly continuous on $[0, \infty)$. Show that there exist constants a and b such that

$$f(x) \leq ax + b, \qquad x \geq 0.$$