Practice Problems for Homework 3

1. Prove by induction that if $m, n \in \mathbb{N}$ then $mn \in \mathbb{N}$.

2. Give a proof of the Modified Induction Principle: If S is a subset of $\mathbb{N}$ that has a smallest number s and such that $x+1 \in S$ whenever $x \in S$ then $S \supseteq \{x \in \mathbb{N} : x > s\}$.

3. Prove by induction that if $n, m \in \mathbb{N}$ and $n > m$ then $n \geq m + 1$.

4. (a) Prove that between any two real numbers there is a rational number.

   (b) Prove that between any two real numbers there is an irrational number. You may use the fact that $\sqrt{2}$ is irrational.

   (c) Prove that any interval contains infinitely many rational numbers and infinitely many irrational numbers.