January 26, 2011 MATH 350 Prof. V. Panferov

Homework Assignment 1

Quiz on Wed. Feb 2, 2011, in class.

1. Let $A = \{x \in \mathbb{N} : 2 < x \leq 6\}$, $B = \{x \in \mathbb{N} : 1 < x < 4\}$, $C = \{x \in \mathbb{N} : x^2 - 4 = 0\}$. Find all elements of the following sets:
   (a) $B \cup C$, (b) $A \cap B \cap C$, (c) $A \cup B \cup C$, (d) $(A \cap B) \cup (B \cap C)$, (e) $B \times C$, (f) $C \times B$.

2. Sets A and B consist of the elements $a = 4n + 2$, $b = 3n$, $n \in \mathbb{N}$. Find $A \cap B$.

3. Find the subsets A and B of the set X if it is known that for every $U \subseteq X$ we have $U \cap A = U \cup B$.

4. Let $f(x) = x$ and $g(x) = \frac{1}{x}$. Is $f = g \circ g$?

5. Find the domain and range for the functions $f(x)$ given by
   (a) $f(x) = \frac{|x|}{x}$, (b) $f(x) = \frac{2}{x^2+1}$, (c) $f(x) = \frac{1}{\sqrt{x-3}}$.

6. Find the domain of $f \circ g$ and $g \circ f$ if $f(x) = \sqrt{x+1}$ and $g(x) = 1/(x-6)$.

7. Let $f : D \to \mathbb{R}$ be defined by $f(x) = (x+1)^2/x$ for $x \in D$ ($D \subseteq \mathbb{R}$ is the largest possible domain of the function). Find D, $f(D)$, $f^{-1}([1,3])$ and $f^{-1}((0,\infty))$.

8. Prove that for non-empty sets A and B, $A \times B = B \times A$ if and only if $A = B$.

9. Prove "De Morgan's laws": for any sets A, B and C

   (a) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

   (b) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

10. Translate the following logical statements from the formal logic notation into plain English. For each of the statements write its negation.

    (a) $\forall x\ ((x \in A) \Rightarrow (x \in B))$

    (b) $\forall x\ ((x \in A) \Leftrightarrow (x \in B))$

    (c) $\forall x\ ((x \in \mathbb{R}) \wedge (x > a) \Rightarrow (f(x) > 0))$

    (d) $\forall \varepsilon > 0\ \exists \delta > 0\ \ \forall x \in \mathbb{R}\ (\,(0 < |x - a| < \delta) \Rightarrow |f(x) - A| < \varepsilon\,)$.

11. Show that for any sets A, B and C

    (a) $A \cup B = A$ if and only if $B \subseteq A$

    (b) $A \setminus B = A$ if and only if $A \cap B = \emptyset$.

12. Let $f : X \to Y$ with $A_1, A_2 \subseteq X$ and $B_1, B_2 \subseteq Y$. Show that

(a) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

(b) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

(c) Give an example when $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$

(d) Show that if f is a one-to-one function then $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$.