Math 350 NAME
Exam 2
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Instructions: Answer all 6 questions. There is one extra credit problem

1) Let $f(x) = \frac{1}{\sqrt{x}}$

a) Show directly (from definition) that $f(x)$ is uniformly continuous on $[\frac{1}{4}, \infty)$

b) Show f is not uniformly continuous on $(0, 1]$ using a Cauchy Sequence (explain)

2) Define $f(x) = \begin{cases} \cos\frac{1}{x} & if\, x \neq 0 \\ 0 & if\, x = 0 \end{cases}$

Prove f is not continuous at 0.

3) Suppose $f : [a,b] \to \mathbb{R}$ is continuous. Prove f must be bounded on $[a,b]$

4) Define $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x^3 & \text{if } x > 1 \end{cases}$

Prove, using the limit definition of derivative, that f is not differentiable at $x = 1$

5) Use the Mean Value Theorem to do a) and b)

a) For all $x,y \in \mathbb{R}$, $|\sin x - \sin y| \le |x - y|$

b) If is differentiable on interval I and $f'(x) = 0$ for all $x \in I$, then f must be constant on I

6) Let $f(x) = \cos x$

a) Use Taylor's Theorem with $a = 0$ to find $P_3(x)$ (that is, the third Taylor Polynomial) and the associated remainder term

b) Find a bound on the error obtained when using $P_3(x)$ to approximate $\cos(.1)$

Extra Credit Problem: Suppose f is differentiable for all real numbers and assume $f(0) = 0$ *and* $f(x + y) \leq f(x) + f(y)$ for all real numbers x *and* y. Use the "h definition of derivative (i.e. where $h \to 0$) to prove $f'(x) \leq f'(0)$ for all real x