#9 (Sec. 5-2, p. 130)

Find $\sqrt{8}$ accurate to 10^{-4}, using Taylor's theorem.

Write

$$\sqrt{8} = \sqrt{9-1} = \sqrt{9\left(1-\frac{1}{9}\right)} = 3\sqrt{1-\frac{1}{9}}$$

$$= 3\left(1 + \frac{1}{2}\left(-\frac{1}{9}\right) + \frac{1}{2!}\,\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{1}{9}\right)^2 + \right.$$

$$\left. + \frac{1}{3!}\,\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{1}{9}\right)^3 + \ldots\right)$$

Using $p_3\left(0, \frac{1}{9}\right)$ as above, find

$$\sqrt{8} \approx 2.8284465020576l3\ldots$$

which has absolute error

$$1.93773 \cdot 10^{-5}.$$

Lagrange's form of the remainder gives

$$\left|\sqrt{8} - 3p_3\left(0, \frac{1}{9}\right)\right| = 3 \cdot \frac{\left|f^{(4)}(c)\right|}{4!}\left(\frac{1}{9}\right)^4$$

$$= \left|3 \cdot \frac{1}{4!}\,\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(1+c)^{-\frac{7}{2}}\left(\frac{1}{9}\right)^4\right|$$

$$\leq \frac{3 \cdot 3 \cdot 5}{4!\,2^4\,9^4} \cdot \left(1-\frac{1}{9}\right)^{-\frac{7}{2}}$$

$$= 2.697400 \cdot 10^{-5}$$