Math 350 NAME
Final
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Instructions: Answer all 8 questions. Show all your work

1) Let $\{a_n\}$ be a convergent sequence with limit L. Prove that $\{a_n\}$ is bounded above and below.

2) Let $a > 0$ be a real number. Use an (ϵ, δ) - argument to prove $\lim_{x \to a} \sqrt{x} = \sqrt{a}$

3) Let I be any interval or real numbers and assume $g : I \to \mathbb{R}$ is differentiable. Assuming $|g'(x)| \leq 3$ for all $x \in I$, prove that g must be uniformly continuous on I.

4) Prove the Product Rule for differentiation. That is, if f and g are differentiable at x_0, then, using the definition of derivative, prove $(fg)'(x_0) = f(x_0)g'(x_0) + g(x_0)f'(x_0)$

5) Let $f(x) = x^{\frac{1}{3}}$ and let $a \neq 0$, a real number. Using the definioin of derivative prove $f'(a) = \frac{1}{3}a^{\frac{-2}{3}}$ (the difference of cubes formula could help).

6) Define $h : [0,2] \to \mathbb{R}$ by $f(x) = \begin{cases} 1 & if\, x \neq 1 \\ 0 & if\, x = 1 \end{cases}$

a) Show $\bar{S}(f;P) = 2$ for every partition P of $[0,2]$.
b) Show f is integrable on $[0,2]$ as follows: for any $\epsilon > 0$, find a partition P_ϵ of $[0,2]$ such that $\underline{S}(f;P_\epsilon) = 2 - \frac{2}{3}\epsilon$. Then combine this with the result in (a).

c) What is the value of $\int_0^2 f$, briefly explain

7) Suppose $f : [a,b] \to \mathbb{R}$ is continuous. Prove f must be Riemann integrable.

8) Suupose $\sum a_n$ and $\sum b_n$ are series of positive terms with $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 1$. Prove

a) If $\sum b_n$ converges, then $\sum a_n$ converges and

b) if $\sum b_n$ diverges, then $\sum a_n$ diverges.