Mon, Jun 8, 2015 MATH 350 Quiz 3

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Given values of L, a and ε determine a value δ such that the statement "$|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$" is valid:

$$f(x) = \frac{\sqrt{2x} - 2}{x - 2}, \quad a = 2, \quad L = \frac{1}{2}, \quad \varepsilon = 0.01.$$

$$\left|\frac{\sqrt{2x}-2}{x-2} - \frac{1}{2}\right| = \left|\frac{\sqrt{2}(\sqrt{x}-\sqrt{2})}{(\sqrt{x}-2)(\sqrt{x}+2)} - \frac{1}{2}\right| = \left|\frac{\sqrt{2}}{\sqrt{x}+2} - \frac{1}{2}\right|$$

$$= \left|\frac{2\sqrt{2} - \sqrt{x} - \sqrt{2}}{2(\sqrt{x}+\sqrt{2})}\right| = \frac{1}{2}\left|\frac{\sqrt{x}-\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right| = \frac{1}{2}\frac{|x-2|}{(\sqrt{x}+\sqrt{2})^2} < \frac{\delta}{4} \le \varepsilon$$

since $|x-2| < \delta$ and $\sqrt{x} \ge 0 \Rightarrow (\sqrt{x}+\sqrt{2})^2 \ge (\sqrt{2})^2 = 2$

Take $\delta = 4\varepsilon = 0.04. \Rightarrow |f(x) - L| < \varepsilon = 0.01.$

OR: Since $\dfrac{\sqrt{2x}-2}{x-2} = \dfrac{\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \dfrac{2}{\sqrt{2x}+2}$ is a decreasing fn. of x,

Solve $\left|\dfrac{2}{\sqrt{2x}+2} - \dfrac{1}{2}\right| \le 0.01$

$\Longleftrightarrow \quad x_1 \le x \le x_2$ where $\dfrac{2}{\sqrt{2x_1}+2} = 0.51$

$\dfrac{2}{\sqrt{2x_2}+2} = 0.49$

$\sqrt{2x_1} + 2 = \dfrac{1}{0.255}$ $\qquad \sqrt{2x_2} + 2 = \dfrac{1}{0.245}$

$x_1 = \dfrac{1}{2}\left(\dfrac{1}{0.255} - 2\right)^2$ $\qquad x_2 = \dfrac{1}{2}\left(\dfrac{1}{0.245} - 2\right)^2$

$x \doteq 1.8462\ldots$ $\qquad y = 2.1666\ldots$

Thus, $\delta = 0.15$ would work...

Please turn over...

2. Prove that for every number a the quadratic function $cx^2 + dx + e$ has the property that

$$\lim_{x\to a}(cx^2 + dx + e) = ca^2 + da + e.$$

$$\lim_{x\to a}(cx^2+dx+e) = \lim_{x\to a} cx^2 + \lim_{x\to a} dx + \lim_{x\to a} e$$

"Lim of a sum"

$$= \lim_{x\to a} c \lim_{x\to a} x \lim_{x\to a} x + \lim_{x\to a} d \lim_{x\to a} x + e$$

"Lim of product" "Lim of const"

$$= ca^2 + da + e.$$

"Lim of const" "Lim of x"

3. Given the set $S = \{x : x^2 - 2x - 3 < 0\}$ find $\sup(S)$ and $\inf(S)$. Justify your answers.

$$x^2 - 2x - 3 = (x-3)(x+1) < 0$$

$$\iff (x > -1) \wedge (x < 3) \iff x \in (-1, 3)$$

$\sup(S) = 3$ upper bd since $\forall x \in S$ $x < 3$

least upper bd. since $\forall s < 3$ $\exists x \in S$, $x > s$.

Indeed, if $s \leq -1$ then any $x \in S$ is OK; if $-1 < s < 3$ then, for instance, $x = \frac{s+3}{2} \in S$, $x < s$.

$\inf(S) = -1$

lower bd. since $\forall x \in S$ $x > -1$

greatest lower bd. since $\forall t > -1$ $\exists x \in S$: $x < t$.

indeed, if $t \geq 3$ then any $x \in S$ will work

if $-1 < t < 3$ then, for instance, $x = \frac{t-1}{2} \in S$, $x > t$.