Wed, Jun 3, 2015 **MATH 350** **Quiz 2**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Prove that if F is a field and $a, b \in F$ then $a(-b) = -(ab)$ and $(-a)(-b) = ab$.

(a) $ab + a(-b) \underset{(\text{distr.})}{=} a(b + (-b)) \underset{(\text{neg.})}{=} a \cdot 0 \underset{(\text{mult by zero})}{=} 0$

$ab + (-ab) = 0 \underset{(\text{uniq. of neg.})}{\Rightarrow} a(-b) = -ab$

(b) $(-a)(-b) - ab \overset{(\text{def. of "}-\text{"})}{=} (-a)(-b) + (-ab)$

$\underset{(\text{part (a)})}{=} (-a)(-b) + a(-b) \underset{(\text{distr., comm})}{=} ((-a) + a)(-b)$

$\underset{(\text{neg.})}{=} 0 \cdot (-b) \underset{(\text{comm.})}{=} (-b)\,0 \underset{(\text{mult. by zero})}{=} 0$

Therefore $(-a)(-b) = ab$.

2. Prove that if $a > b$ and $c < 0$ then $ac < bc$.

$a > b \overset{(\text{def ">"})}{\Rightarrow} a - b > 0$

$c < 0 \overset{(\text{def "<"})}{\Rightarrow} -c > 0$

$\underset{\text{Axiom I}}{\Rightarrow} (a-b)(-c) > 0 \Rightarrow -(a-b)c > 0$

$\Rightarrow (a-b)c < 0$

$\Rightarrow ac - bc < 0$

$\Rightarrow ac < bc$

Please turn over...

3. Prove for any numbers a and b that $|a| - |b| \leq |a - b|$.

$\Longleftrightarrow \quad |a| \leq |a-b| + |b|$ (add $|b|$ to both sides)

$\Longleftrightarrow \quad |(a-b)+b| \leq |a-b| + |b|$ (since $(a-b)+b = a+(-b+b) = a+0 = a$)

Set $A = a-b, \quad B = b$

$|A+B| \leq |A| + |B|$

is true by the triangle inequality.

4. Prove by induction:

$$n, m \in \mathbb{N} \quad \Rightarrow \quad n + m \in \mathbb{N}.$$

If $m = 1$: $\forall n \in \mathbb{N} \quad n+1 \in \mathbb{N}$ since $\mathbb{N}$ is inductive.

For $m \geq 1$ prove by induction:

Set $S = \{m \in \mathbb{N} : \forall n \in \mathbb{N} \quad n+m \in \mathbb{N}\}$

Then $1 \in S$ by the above.

If $k \in S$ then $\forall n \in \mathbb{N} \quad n+k \in \mathbb{N}$

Then $\forall n \quad n+(k+1) = \underbrace{(n+1)}_{\in \mathbb{N} \text{ since } n \in \mathbb{N}} + k \in \mathbb{N}$, since $k \in S$

Therefore $k+1 \in S$

S is inductive $\Rightarrow S = \mathbb{N}$.