Mon, Jun 1, 2015 **MATH 350** **Quiz 1**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Use truth tables to show that for any logical statements P, Q

$$(P \Rightarrow Q) \Leftrightarrow (\overline{Q} \Rightarrow \overline{P}).$$

P \ Q	T	F
T	T	F
F	T	T

$(P \Rightarrow Q)$

$P, \overline{P}$ \ Q, $\overline{Q}$	T, F	F, T
T F	T	F
F T	T	T

$(\overline{Q} \Rightarrow \overline{P})$

Truth tables are identical; the statements are log. equivalent.

2. Prove that for any non-empty sets A and B

$$A \cup B = A \quad \Rightarrow \quad B \subseteq A.$$

WTS $B \subseteq A$; Know: $\underbrace{A \cup B = A}$

$\forall x \quad (x \in B) \Rightarrow (x \in A \cup B) \Rightarrow (x \in A)$

def. of $A \cup B$

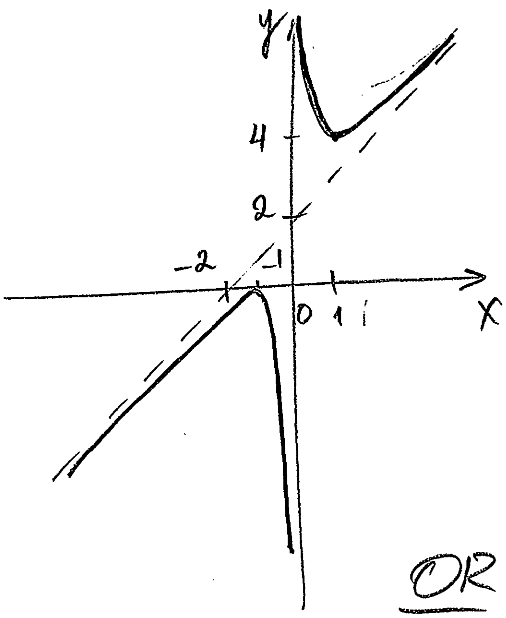
$\Rightarrow \quad B \subseteq A.$

Please turn over...

3. Let $f(x) = (x+1)^2/x$. Find the sets $f(\mathbb{R}\setminus\{0\})$, $f^{-1}((0,\infty))$ and $f^{-1}([1,3])$. Show your work.

Graph: vertical asympt. $x=0$;

$$f(x) = \frac{(x+1)^2}{x} = x+2+\frac{1}{x} \Rightarrow y = x+2 \text{ - oblique as.}$$



$$f'(x) = 1 - \frac{1}{x^2} \Rightarrow \text{crit. pts. } x = \pm 1$$

$$\underbrace{f(1) = 4}_{\text{local min.}}, \quad \underbrace{f(-1) = 0}_{\text{local max.}}$$

$$f(\mathbb{R}\setminus\{0\}) = (-\infty, 0] \cup [4, \infty)$$

$$f^{-1}((0,\infty)) = (0,\infty)$$

$$f^{-1}([1,3]) = \emptyset$$ (there is no x corr. to $1 \le y \le 3$)

OR: Solve $y = \frac{(x+1)^2}{x}$

$$\Longleftrightarrow x = \frac{y-2}{2} \pm \underbrace{\frac{\sqrt{y(y-4)}}{2}}$$

domain gives $f(\mathbb{R}\setminus\{0\})$

$y \ge 4$ produces all $x \ge 0$, etc.

4. Prove or disprove using a counterexample:

For every natural n the number $n^2 + n + 41$ is a prime.

Counterexample: $n = 40$

$$N = 40^2 + 40 + 41 = 41 \cdot 40 + 41 = 41^2$$ – not prime

OR

$$N = 41^2 + 41 + 41 = 43 \cdot 41$$ – not prime

(Curiously, n^2+n+41 is a prime for all $n \le 39$, can't use those values for a counterexample...)