Supplementary Problems for Section 1.4 - Practice

1. Let a, b denote real numbers, and n, m denote natural numbers. Use induction to prove the following:

   (a) If $0 < a < b$ then $a^n < b^n$.

   (b) If $n < m$ and $a > 1$ then $a^n < a^m$.

   (c) If $n < m$ and $0 < a < 1$ then $a^n > a^m$.

2. Prove the binomial theorem: For any a, b real, and any n natural

$$(a+b)^n = \sum_{n=0}^{n} \binom{n}{k} a^k \, b^{n-k}, \quad \text{where} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

3. Prove the formula for the geometric progression:

$$\sum_{k=1}^{n} a^k = \frac{a^{n+1}-1}{a-1}.$$