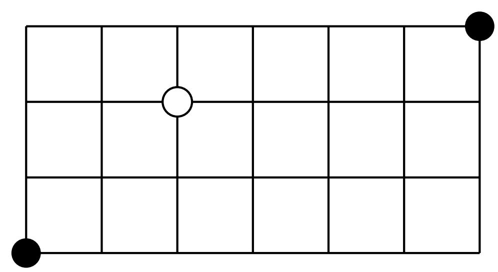
Mar 2, 2014

MATH 340: Midterm 1 review questions

1. **(1.13)** Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place? *[Answer: 190.]*

2. Ten books are placed in random order on a bookshelf. Find the probability of three given books being side-by-side. *[Answer: 1/15.]*

3. Consider the grid of points shown in the figure. Suppose that, starting at the top right corner, you can go one step down or one step left at each move, until you reach the bottom left corner.



   (a) How many different paths from one corner to another are possible?

   (b) How many different paths are there that go through the point marked by the white circle?

4. **(1.30)** Delegates from 10 countries, including Russia, France, England, and the United States, are to be seated in a row. How many different seating arrangements are possible if the French and English delegates are to be seated next to each other, and the Russian and U. S. delegates are not to be next to each other?

5. **(Proposition 1.6.2)** Show that given integer values $n, r > 0$, there are $\binom{n+r-1}{r-1}$ nonnegative integer-valued vectors $(x_1, \ldots, x_r)$ satisfying

$$x_1 + \cdots + x_r = n.$$

6. List the three axioms of probability. Derive, using these axioms, the following properties, true for any events A, B and C in a sample space S:

   (a) $P(A) = P(AB) + P(AB^c)$

   (b) $P(A \cup B) = P(A) + P(B) - P(AB)$

   (c) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$.

7. (a) **(T 2.11)** Show that for any two events A, B in a sample space S

$$P(AB) \geq P(A) + P(B) - 1.$$

   (b) If $P(A) = P(B) = \frac{2}{3}$, show that $P(A|B) \geq \frac{1}{2}$.

8. **(ST 3.1)** In a game of bridge, West has no aces. What is the probability of his partner's having (a) no aces? (b) two or more aces? (c) What would the probabilities be if West had exactly 1 ace?

9. A poker hand is said to be a *flush* if all five cards are of the same suit.

   (a) If five cards are selected at random what is the probability of being dealt a flush?

   (b) What is the conditional probability of being dealt a flush if the first two cards dealt are spades?

10. **(2.33)** A forest contains 20 elk, of which 5 are captured, tagged and then released. A certain time later, 4 of the 20 elk are captured. What is the probability that 2 of these 4 have been tagged?

11. **(2.35)** There are 30 psychiatrists and 24 psychologists attending a certain conference. Three of these 54 people are randomly chosen to take part in a panel discussion. What is the probability that at least one psychologist is chosen?

12. **(2.37)** An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. If a student has figured out 7 of the problems, what is the probability that he or she will answer correctly

    (a) All 5 problems?

    (b) at least 4 of the problems?

13. **(2.38)** There are n socks, three of which are red, in a drawer. What is the value of n such that when two of the socks are chosen randomly, the probability that they are both red is $\frac{1}{2}$?

14. Assume that E and F are two events with positive probabilities. Prove that if $P(E|F) > P(E)$ then $P(F|E) > P(F)$.

15. One urn contains only white balls, while another urn contains 30 white balls and 10 black balls. An urn is selected at random, and then a ball is drawn at random from the urn. The ball turns out to be white, and is then put back into the urn. What is the probability that another ball drawn from the same urn will be black? *[Answer: 3/28.]*

16. Consider n urns, each containing w white balls and b black balls. A ball is drawn at random from the first urn and put into the second urn, then a ball is drawn at random from the second urn and placed into the third urn, and so on, until finally a ball is drawn from the last urn and examined. What is the probability of this ball being white? *[Answer: $w/(w+b)$.]*

17. One coin in a collection of 65 has two heads. The rest are fair. If a coin, chosen at random from a lot and then tossed, turns up heads 6 times in a row, what is the probability that it is a two-headed coin?

18. Prove that the events A and B are independent if $P(B|A) = P(B|A^c)$.

19. **(ST 3.21)** Prove that if $P(A|B) = 1$ then $P(B^c|A^c) = 1$.

20. **(ST 3.30^{8,9}, 3.25^{7})** For any two events E, F show that

$$P(E|E \cup F) \geq P(E|F).$$

Hint: compute $P(E|E \cup F)$ by conditioning on whether F occurs.

21. An urn contains w white balls, b black balls, and r red balls. Find the probability of a white ball being drawn before a black ball, if

    (a) Each ball is replaced after being drawn

    (b) No balls are replaced.

    [Answer: $w/(w+b)$ in both cases.]