Mon. Feb 24, 2014 **MATH 340, Quiz 3** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

* see last page for Alternative solution.

1. Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card is a spade given that the second and the third cards are spades.

$A = \{\text{first spade}\}$, $B = \{\text{2nd spade}\}$, $C = \{\text{3rd spade}\}$

$$P(BC) = P(BC/A)\,P(A) + P(BC/A^c)\,P(A^c)$$

$$= \frac{12}{51}\cdot\frac{11}{50}\;\frac{1}{4} + \frac{13}{51}\cdot\frac{12}{50}\;\frac{3}{4} = \frac{12\cdot 12}{51\cdot 50}$$

$$P(A/BC) = \frac{P(BC/A)\,P(A)}{P(BC)} = \frac{\frac{12\cdot 11}{51\cdot 50}}{\frac{12\cdot 11}{51\cdot 50} + \frac{13\cdot 12\cdot 3}{51\cdot 50}}$$

$$= \frac{11}{11+39} = \frac{11}{50} = \underline{\underline{0.22}}.$$

2. You ask your neigbor to water a sickly plant while you are on vacation. Without water it will die with probablity 0.8; with water it will die with probability 0.15. You are 90 percent certain that your neighbor will remember to water the plant.

(a) What is the probability that the plant will be alive when you return?

$P(D|W) = 0.15$; $P(D|W^c) = 0.8$; $P(W) = 0.9$

$$P(A) = 1 - P(D) = 1 - P(D|W)\,P(W) - P(D|W^c)\,P(W^c)$$

$$= 1 - 0.15\cdot 0.9 - 0.8\cdot 0.1 = 0.785$$

(b) If it is dead, what is the probability that your neighbor forgot to water it?

$$P(W^c|D) = \frac{P(D|W^c)\,P(W^c)}{P(D)}$$

$$= \frac{0.8\cdot 0.1}{1-0.785} \approx 0.372.$$

Please turn over...

3. Consider two independent tosses of a fair coin. Let A be the event that that the first toss results in heads, let B be the event that that the second toss results in heads, and let C be the event that in both tosses the coin lands on the same side. Show that the events A, B, and C are pairwise independent – that is, A and B are independent, A and C are independent, and B and C are independent – but not independent.

$A = \{(h,h), (h,t)\}$ $\qquad P(A) = \frac{1}{2}$

$B = \{(h,h), (t,h)\}$ $\qquad P(B) = \frac{1}{2}$

$C = \{(h,h), (t,t)\}$ $\qquad P(C) = \frac{1}{2}$.

$AB = \{(h,h)\}$

$BC = \{(h,h)\}$ $\qquad P(AB) = P(BC) = P(AC) = \frac{1}{4}$

$AC = \{(h,h)\}$ $\qquad = P(A)P(B) = P(B)P(C) = P(A)P(C)$

$\Rightarrow$ the events are pairwise independent.

$ABC = \{(h,h)\}$

$P(ABC) = \frac{1}{4} \neq \frac{1}{8} = P(A)P(B)P(C)$

$\Rightarrow$ the triple of event A, B, C is not independent.

Another way to see this is to notice that for example

AC implies B $\Rightarrow$

$P(B|AC) = 1 \neq \frac{1}{2} = P(B)$.

The end.

Problem 1: Another way:

Cosider all arrangements of 52 cards in order.

Once we know that 2^{nd} and 3^{rd} cards are spades, we have 50 cards remaining, of which 11 are spades.

Since all arrayements of the remaining 50 cards remain equally likely, there is

$$P = \frac{11}{50} = 0.22$$

probability that the first one is a spade.