Wed. Feb. 10, 2014 **MATH 340, Quiz 2** Prof. V. Panferov

Name: (print) Solutions,

Each problem is worth 2 points. Show all your work.

1. Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is selected randomly, what is the probablity that this student is wearing *a ring and a necklace*?

$R = \{$ student wears a ring $\}$

$N = \{$ student wears a necklace $\}$

$$P(R) = 0.2 \qquad P(N) = 0.3$$

$$P(R^c N^c) = 0.6 \Rightarrow P(R \cup N) = 0.4$$

$$\underbrace{P(R \cup N)}_{0.4} = \underbrace{P(R)}_{0.2} + \underbrace{P(N)}_{0.3} - P(RN)$$

$$\Rightarrow P(RN) = \underline{\underline{0.1}}$$

2. If it is assumed that all $\binom{52}{5}$ cards are equally likely, what is the probablilty of being dealt *a pair*? (This occurs when the cards have denominations a, a, b, c, d where a, b, c and d are distinct.)

$$P(\{\text{"pair"}\}) = \frac{\binom{13}{4}\binom{4}{1}\binom{4}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}} = \frac{\frac{13\cdot12\cdot11\cdot10}{6\cdot4}\cdot 4\cdot 6\cdot 4\cdot 4}{\frac{52\cdot51\cdot50\cdot49\cdot48}{1\cdot2\cdot3\cdot4\cdot5}} = \frac{88}{833}$$

$$\approx \underline{\underline{0.1056}}.$$

Comment:

$\binom{13}{4}$ ways to choose 4 denominations

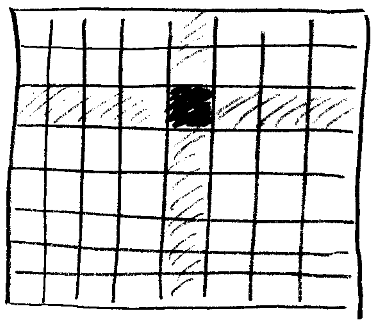
$\binom{4}{1}$ ways to choose denomination "a"

$\binom{4}{2}$ ways to choose suits for "a"

$\binom{4}{1}, \binom{4}{1}$ ways to choose suits for "b" and "c".

Please turn over...

3. If 8 castles (that is, rooks) are randomly places on a chessboard, compute the probablity that none of the rooks can capture any of the others. That is, compute the probability that no row or file contains more than one rook.



8X8 chessboard.

Placing each rook "removes" a row and a file from the chessboard, leaving us with a board of size 7x7, 6x6, etc.

$8 \cdot 8$ ways to place first rook

$7 \cdot 7$ ways to place 2nd rook

$\vdots$

$1 \cdot 1$ way to place 8th rook.

So they do not capture each other.

$64 \cdot 63 \cdots 57$ ways in total to place rooks without restrictions.

$$P = \frac{(8!)^2}{64 \cdot 63 \cdots 57} = \frac{8!}{\binom{64}{8}} \approx 9.109 \cdot 10^{-6}.$$

The end.