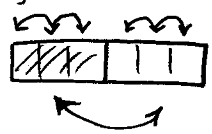
Wed. Jan. 29, 2014 **MATH 340, Quiz 1** Prof. V. Panferov

Name: (print) *Solutions*

Each problem is worth 2 points. Show all your work: give brief *solutions* to the problems.

1. (a) In how many ways can 3 boys and 3 girls sit in a row if the boys and the girls are each to sit together?

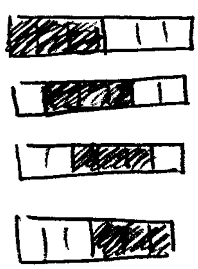
Seat G $\Rightarrow 3!$ ways Seat B $\Rightarrow 3!$ ways



B - G $\Rightarrow 2!$ ways

$2!\,3!\,3! = \underline{\underline{72}}$ ways

(b) In how many ways if only the boys must sit together?



4 ways to seat the boys relative to girls

3! ways to choose seats for boys

3! ways to choose seats for girls

$4 \cdot 3! \cdot 3! = \underline{\underline{144}}$ ways.

2. How many different letter arrangements can be made from the letters

(a) $P_1 R O_1 P_2 O_2 S E$?

7 symbols if distinguish between $P_1, P_2;\ O_1, O_2$

7! ways to arrange $P_1 R O_1 P_2 O_2 S E$

Since permutations of P_1, P_2, O_1, O_2 lead to the same arrangement,

$$\frac{7!}{2!\,2!} = \frac{5040}{4} = \underline{\underline{1260}}.$$

(b) *MISSISSIPPI* ?

11! arrangements of $M I_1 S_1 S_2 I_2 S_3 S_4 I_3 P_1 P_2 I_4$

4! permutations of $I_1 I_2 I_3 I_4$
4! permutations of $S_1 S_2 S_3 S_4$
2! permutations of $P_1 P_2$
lead to the same arrangement.

Please turn over...

$$\frac{11!}{4!\,4!\,2!} = \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot 5\cdot\cancel{6}\cdot 7\cdot\cancel{8}\cdot 9\cdot 10\cdot 11}{\cancel{6}\cdot\cancel{4}\cdot\cancel{4}\cdot\cancel{4}\cdot\cancel{2}} = 35\cdot 990 = 35000 - 350 = \underline{\underline{34{,}650}}.$$

3. Give a combinatorial proof of the fact

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}.$$

Hint: Consider a group of n men and m women. How many groups of size r are possible?

$\binom{n+m}{r}$ = Number of ways to choose a group of r people from $n+m$ people.

$$= N_0 + N_1 + \dots + N_r$$

N_i - number of ways to choose a group of r with i men and $r-i$ women.

$$N_i = \binom{n}{i}\binom{m}{r-i}$$

$$\Rightarrow \binom{n+m}{r} = \sum_{i=0}^{r} \binom{n}{i}\binom{m}{r-i}$$

$$= \binom{n}{0}\binom{n}{r} + \dots + \binom{n}{r}\binom{m}{0}.$$

The end.