Wed. Nov 18, 2015 **MATH 340, Quiz 8** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Let X be a normal random variable with mean 12 and variance 4. Find the value c such that $P(X > c) = 0.1$.

$\sigma^2 = 4 \Rightarrow \sigma = 2$

$$P(X>c) = 0.1$$

$$P\left(\frac{X-12}{2} > \frac{c-12}{2}\right) = 0.1$$

$$P\left(Z > \frac{c-12}{2}\right) = 0.1$$

$$P\left(Z \le \frac{c-12}{2}\right) = 0.9$$

$$\frac{c-12}{2} \simeq 1.28$$

$$c \simeq 12 + 2 \cdot 1.28$$

$$c \simeq 14.56$$

2. Each item produced by a certain manufacturer is, independently, of acceptable quality with probability 0.95. Approximate the probability that at most 10 of the next 150 items produced are unacceptable.

$$X \sim \text{Binomial}\,(n=150,\ p=0.05) \approx \text{Normal}\,(\mu = 7.5,\ \sigma^2 = 7.125)$$

$$P(X \le 10) = P(X \le 10.5) = P\left(\frac{X-7.5}{\sqrt{7.125}} \le \frac{10.5-7.5}{\sqrt{7.125}}\right)$$

$$\approx P\left(Z \le \frac{3}{\sqrt{7.125}}\right) \approx P(Z \le 1.1239)$$

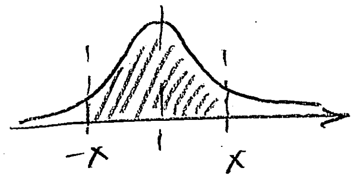
$$= \Phi(1.1239) \approx 0.8686$$

Please turn over...

3. Show that if Z is a standard normal random variable then, for $x > 0$,

$$P(|Z| < x) = 2P(Z < x) - 1.$$

Geometric way:



$$P(|Z| \geq x) = 2P(Z \geq x)$$
$$1 - P(|Z| < x) = 2(1 - P(Z < x))$$
$$P(|Z| < x) = 2P(Z < x) - 1$$

Via integrals:

$$\int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt - \int_{-\infty}^{-x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$
$$= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt - \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 2\int_{-\infty}^{x} + \int_{-\infty}^{x} - \int_{x}^{\infty}$$
$$= 2\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$
$$= 2\Phi(x) - 1.$$

4. If X is uniformly distributed over $(0, 1)$, find the density function of $Y = e^X$.

$$F_Y(a) = P(Y \leq a) = P(e^X \leq a)$$
$$= \begin{cases} 0, & a < 1 \\ P(X \leq \ln a), & a > 1 \end{cases} = \begin{cases} 0, & a < 1 \\ \ln a, & 1 \leq a < e \\ 1, & a \geq e. \end{cases}$$

$$f_Y(a) = \frac{d}{da} F_Y(a) = \begin{cases} 0, & a < 1 \\ \frac{1}{a}, & 1 < a < e \\ 0, & a > e. \end{cases}$$