Wed. Oct 28, 2015 **MATH 340, Quiz 7** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. The probability density function of X, the lifetime of a certain type of elctronic device (measured in hours), is given by

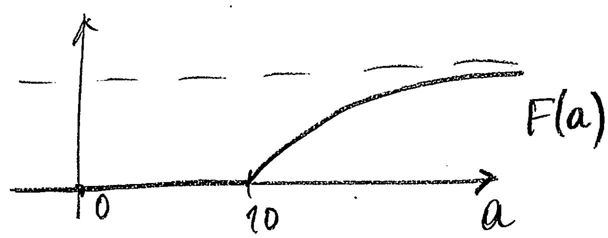
$$f(x) = \begin{cases} \frac{10}{x^2}, & x > 10 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $P(X > 20)$. (b) Find the cumulative distribution function of X.

$$(a) \quad P(X>20) = \int_{20}^{\infty} \frac{10}{x^2}\,dx = \left[-\frac{10}{x}\right]_{20}^{\infty} = 0 + \frac{1}{2} = \frac{1}{2}.$$

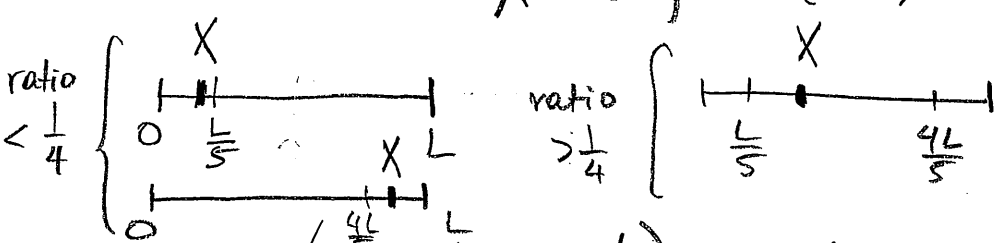
$$(b) \quad F(a) = P(X \le a) = \int_{-\infty}^{a} f(x)\,dx = \begin{cases} 0, & a \le 10 \\ \int_{10}^{a} \frac{10}{x^2}\,dx, & a > 10 \end{cases}$$

$$= \begin{cases} 0, & a \le 10 \\ 1 - \frac{10}{a}, & a > 10 \end{cases}$$

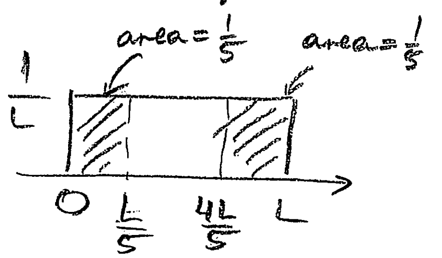


2. A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $\frac{1}{4}$.

$X \sim \text{Uniform}(0, L)$ – equal probability to be anywhere in the interval; (for any subinterval of positive length)



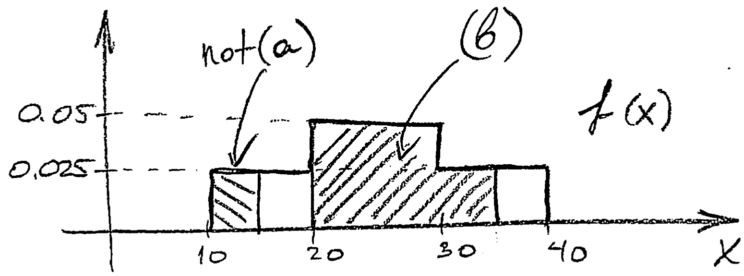
$$P\left(\frac{\text{shorter}}{\text{longer}} < \frac{1}{4}\right) = P\left(X < \frac{L}{5} \text{ or } X > \frac{4L}{5}\right)$$



$$= \frac{\frac{L}{5} + \frac{L}{5}}{L} = \frac{2}{5}.$$

Please turn over...

3. The number of minutes of playing time of a certain high school basketball player in a randomly chosen game is a random variable whose probability density is given in the following figure:



Find the probability that the player plays

(a) more than 15 minutes

(b) between 20 and 35 minutes.

(a) $P(X>15) = 1 - P(X<15) = 1 - 5 \cdot 0.025$

$= 1 - 0.125 = 0.875$

(b) $P(20 < X < 35) = 10 \cdot 0.05 + 5 \cdot 0.025$

$= 0.5 + 0.125 = 0.625$