Wed. Oct 21, 2015 **MATH 340, Quiz 6** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. The number of times that a person contracts a cold is a Poisson random variable with parameter $\lambda = 5$. Suppose a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 3$ for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect. If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial for him or her?

B – event that the drug is beneficial

$P(B) = 0.75 \quad P(B^c) = 0.25$

X – number of colds in a year

$$P(X=2 \mid B) = \frac{3^2}{2!} e^{-3} = \frac{9}{2} e^{-3}$$

$$P(X=2 \mid B^c) = \frac{5^2}{2!} e^{-5} = \frac{25}{2} e^{-5}$$

$$P(B \mid X=2) = \frac{P(X=2 \mid B) P(B)}{P(X=2 \mid B) P(B) + P(X=2 \mid B^c) P(B^c)}$$

$$= \frac{\frac{9}{2} e^{-3} \cdot \frac{3}{4}}{\frac{9}{2} e^{-3} \cdot \frac{3}{4} + \frac{25}{2} e^{-5} \cdot \frac{1}{4}}$$

$$= \frac{1}{1 + \frac{25}{27} e^{-2}} \approx 0.88864$$

Please turn over...

2. On average, 5.2 hurricanes hit a certain region in a year. What is the probability that there will be 3 or fewer hurricanes hitting this year? *Justify the assumptions you are making!*

Assume that occurence of hurricanes is modeled by a Poisson random var. (During each period of time, say 1 day, there is a small prob. of hurricane forming, independent of what happened before.) Rmk: this assumption is clearly not realistic, but we'll give it our best shot.

$X \sim \text{Poisson}(\lambda = 5.2)$

$$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$
$$= e^{-5.2}\left(1 + 5.2 + \frac{(5.2)^2}{2} + \frac{(5.2)^3}{6}\right) \approx 0.2380$$

3. If X is a Binomial random variable with expected value 6 and variance 2.4, find $P(X = 5)$.

$X \sim \text{Binomial}(n,p)$; $np = 6$; $np(1-p) = 2.4$;

$$\Rightarrow \frac{np(1-p)}{np} = 1-p = \frac{2.4}{6.0} = \frac{2}{5} = 0.4$$

$$\Rightarrow p = 1-(1-p) = 0.6$$

$$\Rightarrow n = 10$$

$$P(X=5) = \binom{10}{5}\, 0.6^5 \cdot 0.4^5 = 252 \cdot (0.24)^5$$

$$\approx 0.20$$