Wed. Oct 14, 2015 **MATH 340, Quiz 5** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. If the cumulative distribution function of X is given by

$$F(a) = \begin{cases} 0, & b < 0 \\ \frac{1}{2}, & 0 \le a < 1 \\ \frac{3}{5}, & 1 \le a < 2 \\ \frac{4}{5}, & 2 \le a < 3 \\ \frac{9}{10}, & 3 \le a < 3.5 \\ 1, & a \ge 3.5 \end{cases}$$

calculate the probability mass function of X. *[Graph a bar diagram for the PMF.]*

$X \in \{0, 1, 2, 3, 3.5\}$

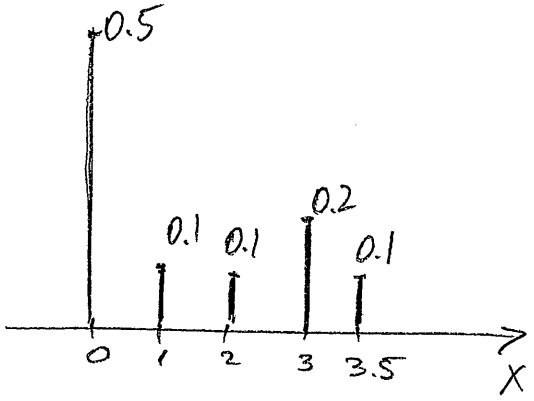
$P(X=0) = \frac{1}{2} = 0.5$

$P(X=1) = \frac{3}{5} - \frac{1}{2} = 0.1$

$P(X=2) = \frac{4}{5} - \frac{3}{5} = 0.2$

$P(X=3) = \frac{9}{10} - \frac{4}{5} = 0.1$

$P(X=3.5) = 1 - \frac{9}{10} = 0.1$



2. Suppose $P(X = 0) = 1 - P(X = 1)$. If $\mathbb{E}[X] = 3\mathrm{Var}(X)$, find $P(X = 0)$.

$P(X=0) = p$, then $P(X=1) = 1-p$.

$\mathbb{E}[X] = 0 \cdot p + 1 \cdot (1-p) = 1-p$

$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = (1-p) - (1-p)^2 = p(1-p)$

$1-p = 3p(1-p)$

$p=1$ or $3p=1 \Leftrightarrow p = \frac{1}{3}$

Please turn over...

3. It is known that diskettes produced by a certain company will be defective with probability 0.01, independently of one another. The company sells the diskettes in packages of 10 and offers a money-back guarantee that at most 1 of the 10 diskettes in the package will be defective. If someone buys 3 packages, what is the probability that exactly one of the packages will be returned under the money-back guarantee?

X – number of defective diskettes in a package of 10.

$X \sim \text{Binomial}(n=10, p=0.01)$

$P(\text{"package returned"}) = P(X \geq 2)$

$= 1 - P(X=0) - P(X=1) = 1 - 0.99^{10} - 10 \cdot 0.99^{9} \cdot 0.01$

$\approx 4.2662 \cdot 10^{-3}$

Y – number of packages returned

$Y \sim \text{Binomial}(n=3, p=4.2662 \cdot 10^{-3})$

$P(Y=1) = \binom{3}{1} p(1-p) = 3p(1-p) \approx 0.012744$