Wed. Sept 30, 2015 **MATH 340, Quiz 4** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

$$P(w/U_A) = \frac{5}{12} \qquad P(U_A) = \frac{1}{2}$$

$$P(w/U_B) = \frac{3}{15} \qquad P(U_B) = \frac{1}{2}$$

fair coin

$$P(U_B/w) = \frac{P(w/U_B)\,P(U_B)}{P(w/U_B)\,P(U_B) + P(w|U_A)\,P(U_A)}$$

$$= \frac{\frac{3}{15}\cdot\frac{1}{2}}{\frac{3}{15}\cdot\frac{1}{2} + \frac{5}{12}\cdot\frac{1}{2}} = \frac{3}{3 + 5\cdot 15/12}$$

$$= \frac{36}{111} \approx 0.3243$$

Please turn over...

2. Suppose that each child born to a couple is equally likely to be a boy or a girl, independently of the sex distribution of the other children in the family. For a couple having 5 children, compute probabilities of the following:

   (a) All children are of the same sex.

   (b) Exactly 3 of the children are boys.

   (c) There is at least one girl.

(a) $P(GGGGG) + P(BBBBB) = \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5 = \frac{1}{16}$

(b) $P(\text{"3 boys"}) = \binom{5}{2}\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2 = \frac{10}{32} = \frac{5}{16}$

↑ ways to choose a sequence with 3 boys, 2 girls

(c) $P(\text{"at least one girl"}) = 1 - P(BBBBB)$

$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$

3. In each of n independent tosses of a coin, the coin lands on heads with probability p. How large need n be so that the probability of obtaining at least one head is at least $\frac{1}{2}$?

$P(\text{"at least one heads"}) = 1 - P(\text{"all tails"})$

$= 1 - (1-p)^n \geq \frac{1}{2}$

$(1-p)^n \leq \frac{1}{2}$

$n \log(1-p) \leq \log \frac{1}{2}$

$n \geq \dfrac{\log \frac{1}{2}}{\log(1-p)}$ (since $1-p < 1 \Rightarrow \log(1-p) < 0$)