Mon. Sept 14, 2014 MATH 340, Quiz 2 Prof. V. Panferov

Name: (print) *Solutions.*

Each problem is worth 2 points. Show all your work.

1. (a) In an experiment, die is rolled repeatedly until a 6 appears, at which point the experiment stops. Describe the sample space of this experiment.

   (b) Let E_n denote the event that n rolls are necessary to complete the experiment. What points of the sample space are in E_n? What is $\left(\cup_{n=1}^{\infty} E_n\right)^c$?

(a) $S = \{(6), (x_1, 6), (x_1, x_2, 6), \ldots, (x_1, x_2, \ldots) : x_i \in \{1..5\}\}$

set of all sequences of rolls ending with "6" + the outcome $(x_1, x_2, \ldots)$ "6 never appears".

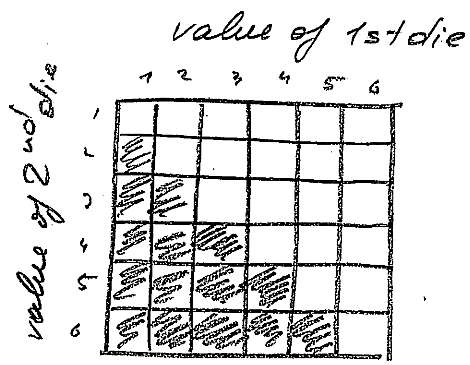
More formally: $S = \{(x_1 .. x_k, 6) : k = 1, 2, .., x_i \in \{1..5\}\} \cup \{(6)\} \cup \{(x_1, x_2 \ldots)\}$.

(b) $E_n = \{(x_1 .. x_{n-1}, 6) : x_i \in \{1..5\}\}$

– set of all sequences of length n ending with a "6";

$\left(\bigcup_{n=1}^{\infty} E_n\right)^c = \bigcap_{n=1}^{\infty} E_n^c = \{(x_1, x_2 \ldots)\} =$ "6 is never rolled".

2. A pair of fair dice is rolled. Find the probability that the second die lands on a higher value than does the first.



$P(E) = \frac{15}{36} = \frac{5}{12} = 0.41\overline{6}$

(≈ 0.42)

Rmk: Out of 36 possible outcomes 6 correspond to equal values of the dice; of the remaining 30, half corresponds to the event $\{2^{nd}$ die $> 1^{st}$ die$\}$

$\Rightarrow \frac{15}{36}$.

Please turn over...

poker hands

3. If it is assumed that all $\binom{52}{5}$ ~~cards~~ are equally likely, what is the probablilty of being dealt *a pair*? (This occurs when the cards have denominations a, a, b, c, d where a, b, c and d are distinct.)

One solution: 13 ways to choose den. "a"

$\binom{4}{2}=6$ ways to choose suits for card "a"

$\binom{12}{3}$ ways to choose denominations "b", "c", "d"

$\binom{4}{1}\cdot\binom{4}{1}\cdot\binom{4}{1}=4^3$ ways to choose suits for "b", "c", "d".

Basic princ. of counting + "Classical definition of probability"

$$\Rightarrow P(E)=\frac{13\cdot 6\cdot \frac{12\cdot 11\cdot 10}{1\cdot 2\cdot 3}\cdot 4^3}{\binom{52}{5}}=\frac{1056}{2499}\approx 0.42$$

4. Prove the following relations: $F = FE \cup FE^c$ and $E \cup F = E \cup E^cF$.

One solution: Use formal rules from Section 2.2

$$F = FS = F(E\cup E^c) = FE \cup FE^c.$$

$$\begin{aligned} E\cup F &= (E\cup E^c)(E\cup F) = E(E\cup F)\cup E^c(E\cup F) \\ &= (EE\cup EF)\cup(E^cE\cup E^cF) \\ &= E \cup E^cF \quad (\text{since } E^cE=\emptyset) \\ &\quad (\text{since } EE=E \text{ and } EF\subseteq E) \end{aligned}$$

Another solution: Use formal logic!

Let $x\in F$ then either $x\in E \Rightarrow x\in EF$ or $x\in E^c \Rightarrow x\in FE^c$.

$\Rightarrow x\in EF\cup FE^c$

Let $x\in FE\cup FE^c$ then $x\in FE$ or $x\in FE^c$. In both cases $x\in F$

Let $x\in E\cup F$ then $x\in E$ or $x\in F \Rightarrow x\in E$ or $x\in EF$ or $x\in E^cF$

$\Rightarrow x\in E$ or $x\in E^cF$.

Since $E^cF\subseteq F$, $E\cup E^cF\subseteq E\cup F \Rightarrow E\cup E^cF = E\cup F$. The end.

Third solution: Use Venn diagrams!