Wed Sept 2, 2015 MATH 340, Quiz 1 Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. How many linear arrangements are there of the letters A, B, C, D, E, F for which

   (a) A is before B, and B is before C?

   Among the $6! = 720$ linear arrangements of the letters A, B, C, D, E, F there are as many of those where A, B, C appear in the original order, as there are with any other of the $3! = 6$ arrangements of A, B, C.

   $\Rightarrow$ $720/6 = 180$ arrangements have A before B, B before C.

   (b) E is not the last in line?

   Among the $6! = 720$ arrangements there are $5! = 120$ of those where E is last in line.

   $\Rightarrow$ $720 - 120 = 600$ arrangements do not have E as last in line.

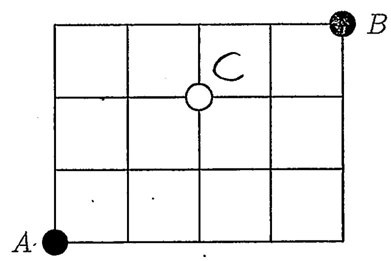
2. A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

$$\underbrace{\binom{12}{5}}_{\text{ways to choose men}}\overbrace{\binom{10}{5}}^{\text{ways to choose women}}\cdot \underbrace{5!}_{\text{ways to pair 5 men with 5 women (permutations of 5 objects)}} = \frac{\cancel{12}\cdot 11\cdot \cancel{10}\cdot 9\cdot 8}{1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{5}}\cdot\frac{\cancel{10}\cdot 9\cdot 8\cdot 7\cdot 6}{1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{5}}\cdot \cancel{120}^{\,10}$$

$$= 11\cdot 9\cdot 8\cdot 9\cdot 8\cdot 7\cdot 6\cdot 10 = 23{,}950{,}080$$

Please turn over...

3. How many different paths are there from A to B that go through the point circled in the following lattice?



$A \to C$: two steps up, two steps right $\binom{4}{2} = 6$ ways

$C \longrightarrow B$: one step up, two steps right $\Rightarrow \binom{3}{1} = 3$ ways

B.P.C $\Rightarrow$ $6 \cdot 3 = 18$ ways.

4. Use the Binomial Theorem to show that for $n > 0$,

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0.$$

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

Use $x = -1$, $y = 1$:

$$0 = (-1+1)^n = \sum_{i=0}^{n} \binom{n}{i} (-1)^i 1^{n-i}$$

$$= \sum_{i=0}^{n} \binom{n}{i} (-1)^i.$$