Wed, May 15, 2013 **MATH 340, Final Exam** Prof. V. Panferov

Name: (print) _____

CSUN ID No. : _____

This exam includes 10 questions in the main part (54 points), and one bonus problem (6 extra points). The exam contains 10 pages; the last page has a table of the standard normal distribution. The duration of the exam is 2 hours.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	8	9	10	11	total

Important: The exam is closed books/notes. Graphing calculators are not permitted. Show all your work.

1. (4 points) A forest contains 20 elk, of which 5 are captured, tagged and then released. A certain time later, 4 of the 20 elk are captured. What is the probability that 2 of these 4 have been tagged? [Simplify your answer to a fraction or a decimal.]

2. (6 points) (a) Assume that E and F are two events with positive probabilities. Show that if $P(E|F) = P(E)$ then $P(F|E) = P(F)$.

(b) If E and F are two events with positive probabilities, and $P(E) < 1$ show that

$$P(E|F) = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}$$

Continued...

3. (6 points) One coin in a collection of 50 has two heads. The rest are fair. If a coin, chosen at random from a lot and then tossed, turns up heads 5 times in a row, what is the probability that it is a two-headed coin?

4. (6 points) The annual rainfall (in inches) in a certain region is normally distributed with parameters $\mu = 40$ and $\sigma = 5$.

   (a) What is the probability that the annual rainfall exceeds 50 inches in a given year?

   (b) What is the probability that starting this year, it will take more than 5 years before a year with annual rainfall of 50 inches?

Continued...

5. (6 points) The cumulative distribution function of X is given by

$$F(a) = \begin{cases} 0, & a < 0 \\ a, & 0 \leq a \leq 1 \\ 1, & a > 1. \end{cases}$$

   (a) Find the probability density of X.

   (b) Find $\mathbb{E}[X]$.

   (c) Find $\mathrm{Var}(X)$.

Continued...

6. (6 points) If 50 percent of the population of a large community is in favor of a proposed rise in sales taxes, approximate the probability that a random sample of 100 people will contain

   (a) at least 60 who are in favor of the proposition;

   (b) between 30 and 40 inclusive who are in favor.

   [Hint: Use the normal approximation with the 0.5 *continuity correction.]*

Continued...

7. (6 points) The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} c/y, & 0 < y < 1,\ 0 < x < y \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Determine the value of the constant c.

   (b) Find $\mathbb{E}[XY]$.

Continued...

8. (6 points) Jack and Joe agree to meet at a certain location at about 10 : 30 am. Jack arrives at a time uniformly distributed between 10 : 20 and 10 : 45 and Joe independently arrives at a time uniformly distributed between 10 : 15 and 10 : 40.

   (a) Find the probability that Jack arrives first.

   (b) What is the probability that the first to arrive waits no longer than 5 minutes?

Continued...

9. (4 points) (a) State Chebyshev's inequality for a random variable X with mean μ and variance σ^2.

   (b) If X_i are independent, identically distributed random variables, $\mathbb{E}[X_i] = 5$, and $\mathrm{Var}(X_i) = 1$, estimate the probability

$$P\Big(\Big|\frac{X_1 + \cdots + X_{10}}{10} - 5\Big| > 0.1\Big).$$

10. (4 points) Use the Central Limit Theorem to estimate the probability that among $10,000$ random digits the digit 3 appears not more than 931 times.

Continued...

11. (bonus: 6 points) Let U be uniformly distributed on $(0, 1)$. What is the probability that the equation

$$x^2 + 4Ux + 1 = 0$$

has two real roots, x_1 and x_2?

The end.