

Name: (print) _____

Solutions

Each problem is worth 2 points. Show all your work.

1. Use the determinant to find out for which values of the constant k the matrix $A = \begin{pmatrix} k & 2 \\ 3 & 4 \end{pmatrix}$ is invertible.

$$\det(A) = 4k - 6 = 0$$

$$\Rightarrow k = \frac{3}{2}$$

A is not invertible if $k = \frac{3}{2}$

A is invertible if $k \neq \frac{3}{2}$.

2. Find the determinants

$$(a) \begin{vmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{vmatrix}$$

$$(b) \begin{vmatrix} 2 & 5 & 7 & 11 \\ 0 & 3 & 5 & 13 \\ 0 & 0 & 5 & 11 \\ 0 & 0 & 0 & 7 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 4 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{vmatrix}$$

$$= -1 \cdot 2 \cdot 3 \cdot 4 = -24.$$

$$= 2 \cdot 3 \cdot 5 \cdot 7 = 210.$$

Please turn over...

3. (2 points) Consider a 4×4 matrix A with rows v_1, v_2, v_3, v_4 ; written symbolically,

$$A = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix}. \quad \text{If } \det(A) = 8, \text{ find}$$

$$(a) \quad \det \begin{pmatrix} v_1 \\ v_2 \\ -9v_3 \\ v_4 \end{pmatrix}$$

$$= -9 \det(A) \\ = -72.$$

$$(b) \quad \det \begin{pmatrix} v_2 \\ v_3 \\ v_1 \\ v_4 \end{pmatrix}.$$

$$= - \det \begin{pmatrix} v_2 \\ v_3 \\ v_4 \\ v_1 \end{pmatrix} \\ = \det \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 8.$$

4. (a) If A is an $n \times n$ matrix and $\det(A) = 5$ find $\det(-A)$.

$$(-1)^n \cdot 5$$

$$\left(\begin{array}{l} -1 \text{ factors out from } n \text{ rows} \\ \text{(or } n \text{ columns)} \end{array} \right).$$

- (b) If $\det(A) = 3$ for some $n \times n$ matrix what is $\det(A^T A)$?

$$\det(A^T A) = \det(A^T) \det(A) \\ = (\det(A))^2 = 9.$$