Wed, Nov 16, 2009 **MATH 262** **Quiz 4**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Consider a linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$. We are told that the matrix of T with respect to the basis $\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}$ is $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$. Find the standard matrix of T (in the basis $\vec{e}_1$, $\vec{e}_2$) in terms of a, b, c, and d.

$\left[A\begin{pmatrix}0\\1\end{pmatrix}\right]_{\mathcal{B}} = \begin{pmatrix}a\\c\end{pmatrix}$; $\left[A\begin{pmatrix}1\\0\end{pmatrix}\right]_{\mathcal{B}} = \begin{pmatrix}b\\d\end{pmatrix}$

$\mathcal{B} = \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}$

$A\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}c\\a\end{pmatrix}$ (second column) ; $A\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}d\\b\end{pmatrix}$ (first column)

$A = \begin{pmatrix}d & c\\ b & a\end{pmatrix}$

2. Which of the following are the subspaces of $\mathbb{R}^{3\times 3}$. (State your answer only: yes/no.)

   (a) The invertible 3×3 matrices no.

   (b) The diagonal 3×3 matrices yes.

   (c) The upper triangular 3×3 matrices yes.

   (d) The 3×3 matrices in reduced row-echelon form no.

Please turn over...

3. Find a basis in the space of all lower-triangular 2×2 matrices, and determine its dimension.

$$\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

span all lower-triangular matrices; linearly independent $\Rightarrow$ basis.

4. Find the orthogonal projection of the vector $9\vec{e}_1$ onto the subspace of $\mathbb{R}^4$ spanned by

$$V_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad V_2 = \begin{pmatrix} -2 \\ 2 \\ 0 \\ 1 \end{pmatrix}. \qquad 9e_1 = \begin{pmatrix} 9 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(9e_1 \cdot V_1) = 18, \qquad (9e_1 \cdot V_2) = -18$$

$$\| V_1 \| = \sqrt{2^2 + 2^2 + 1} = 3 \qquad \| V_2 \| = \sqrt{(-2)^2 + 2^2 + 1} = 3$$

$$\operatorname{proj}_V (9e_1) = \frac{(9e_1 \cdot V_1)}{\|V_1\|^2} V_1 + \frac{(9e_2 \cdot V_2)}{\|V_2\|^2} V_2$$

$$= 2\begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \end{pmatrix} - 2\begin{pmatrix} -2 \\ 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 2 \\ -2 \end{pmatrix}.$$