Wed., Oct 7 2009 **MATH 262** **Quiz 2**

Name: (print) Solutions

Each problem is worth 2 points. Show all your work.

1. Find the inverses if they exist:

(a) $\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}^{-1}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$

$$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 5 & 8 & 0 & 1 \end{array}\right) \cdot \tfrac{5}{2}$$

$$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right) \quad 2$$

$$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 0 & 1 & -5 & 2 \end{array}\right) \cdot 3$$

$$\left(\begin{array}{cc|cc} 2 & 0 & 16 & -6 \\ 0 & 1 & -5 & 2 \end{array}\right) \div 2$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 8 & -3 \\ 0 & 1 & -5 & 2 \end{array}\right)$$

or, using

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

then directly

$$\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}^{-1} = \frac{1}{16-15}\begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix}}}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad -2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad -2 \quad -3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

2. Find the product: $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} = \begin{pmatrix} g & h & k \end{pmatrix}$$

$$\begin{pmatrix} g & h & k \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \underline{\underline{h}}.$$

Please turn over...

3. For the following matrices find A^2, A^3, and A^{1001}.

(a) $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^3 = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\vdots$$

$$A^k = (-1)^k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^{1001} = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A.$$

$$A^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\vdots$$

$$A^k = \begin{cases} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & k \text{ odd} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & k \text{ even} \end{cases}$$

$$A^{1001} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A.$$

4. Consider the transformation T from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

Is this transformation linear? If so, find its matrix.

$$x_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Ax$$

The transformation is linear (matrix times a vector!)

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}.$$