Wed, Sep. 30, 2009 **MATH 262** **Quiz 1**

Name: (print) Solutions

Each problem is worth 2 points. Do not just state the answers, justify them! Show all work.

1. Find all solutions of the system

$$\begin{cases} 7x - y = \lambda x \\ -6x + 8y = \lambda y \end{cases}$$

for (a) $\lambda = 10$ and (b) $\lambda = 15$.

$\lambda = 10$: $\begin{cases} -3x - y = 0 \\ -6x - 2y = 0 \end{cases}$

$3x + y = 0$

$\begin{cases} x = -t/3 \\ y = t \end{cases}$, t - free

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1/3 \\ 1 \end{pmatrix} t$

$\lambda = 15$: $\begin{cases} -8x - y = 0 \\ -6x - 7y = 0 \end{cases}$

$\left(\begin{array}{cc|c} -8 & -1 & 0 \\ -6 & -7 & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} -8 & -1 & 0 \\ 0 & -6.25 & 0 \end{array}\right)$

$\begin{cases} x = 0 \\ y = 0 \end{cases}$

2. Consider a linear system $Ax = b$ where A is a 4×3 matrix. We are told that the rank of the augmented matrix $(A \mid b)$ is 4. How many solutions does the system have?

$$(A|b) = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \\ a_{41} & a_{42} & a_{43} & b_4 \end{array}\right)$$

rank = 4 $\Rightarrow$ reduces to

$$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow \text{inconsistent (no solution.)}$$

Please turn over...

3. Compute the products if they are defined

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \end{pmatrix}$

not defined

matrix has 3 columns,

vector has 2 components.

(b) $\begin{pmatrix} 1 & 1 & -1 \\ -5 & 1 & 1 \\ 1 & -5 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$1 + 2 - 3 = 0$

$-5 + 2 + 3 = 0$

$1 - 10 + 9 = 0$

4. Let $x = \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix}$ and $y = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. Find a *diagonal* matrix A such that $Ax = y$.

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$5a = 2, \quad 3b = 0, \quad -9c = 1$

$a = \frac{2}{5}, \quad b = 0, \quad c = -\frac{1}{9}$

$$\begin{pmatrix} 2/5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1/9 \end{pmatrix}.$$