Wed., Dec. 3, 2008 **MATH 262** **Quiz 5**

Name: (print) Solutions

Show all your work.

1. (2 points) Is there an orthogonal transformation T from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that

$$T\begin{pmatrix}2\\3\\0\end{pmatrix}=\begin{pmatrix}3\\0\\2\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}-3\\2\\0\end{pmatrix}=\begin{pmatrix}2\\-3\\0\end{pmatrix}? \qquad \text{Explain.}$$

No. Orthogonal transformations preserve the dot product. Here we have

$$\begin{pmatrix}2\\3\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\2\\0\end{pmatrix}=0$$

$$\#$$

$$\begin{pmatrix}3\\0\\2\end{pmatrix}\cdot\begin{pmatrix}2\\-3\\0\end{pmatrix}=6$$

2. (2 points) Consider the orthonormal vectors u_1, u_2, u_3, u_4, u_5 in $\mathbb{R}^{10}$. Find the length of the vector

$$x = 7u_1 - 3u_2 + 2u_3 + u_4 - u_5.$$

Pythagorean theorem.

$$\|x\|^2 = \|7u_1\|^2 + \|3u_2\|^2 + \|2u_3\|^2 + \|u_4\|^2 + \|u_5\|^2$$

$$= 49\|u_1\|^2 + 9\|u_2\|^2 + 4\|u_3\|^2 + \|u_4\|^2 + \|u_5\|^2$$

$$= 49+9+4+1+1 = 64$$

$$\Rightarrow \|x\| = 8.$$

Please turn over

3. (2 points) Perform the Gram-Schmidt process on the columns of the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}; \quad \|v_1\| = 2; \quad u_1 = \frac{v_1}{\|v_1\|} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

$$u_2' = v_2 - (v_2 \cdot u_1)u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{2}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\right)\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\|u_2'\| = 1, \quad u_2 = \frac{u_2'}{\|u_2'\|} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}.$$

$$u_3' = v_3 - (v_3 \cdot u_1)u_1 - (v_3 \cdot u_2)u_2$$

$$= \begin{pmatrix} 0 \\ 2 \\ 1 \\ -1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = u_3$$

[QR - factorization is

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$]