Fri., Nov. 7, 2008 **MATH 262** **Quiz 4**

Name: (print) Solutions.

Show all your work.

1. (2 points) Do all invertible 3×3 matrices form a subspace of $\mathbb{R}^{3\times 3}$? Explain.

No. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ are invertible, but their sum, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is not.

2. (2 points) Consider the transformation $T(f(t)) = f''(t) + 4f'(t)$ from P_2 to P_2. Is it linear? Is it an isomorphism? Justify your answers.

Linear.

$$T(f(t)+g(t)) = (f(t)+g(t))'' + 4(f(t)+g(t))' = (f''(t) + 4f'(t)) + (g''(t)+4g'(t)) = T(f(t)) + T(g(t))$$

$$T(kf(t)) = kf''(t) + 4kf'(t) = k(f''(t)+4f'(t)) = k\,T(f(t)).$$

Not an isomorphism.

$$T(1) = 0 \Rightarrow 1 \in \ker(T) \Rightarrow \ker(T) \neq \{0\}$$

$\Rightarrow$ not an isomorphism.

Please turn over...

3. (2 points) Are the polynomials $f(t) = 1 + 4t + 7t^2$, $g(t) = 2 + 5t + 8t^2$, $h(t) = 3 + 6t + 9t^2$ linearly independent?

Use coordinates: Basis $\{1, t, t^2\}$

$$f(t) \to \underbrace{\begin{pmatrix}1\\4\\7\end{pmatrix}}_{= v_1} \quad g(t) \to \underbrace{\begin{pmatrix}2\\5\\8\end{pmatrix}}_{= v_2} \quad h(t) \to \underbrace{\begin{pmatrix}3\\6\\9\end{pmatrix}}_{= v_3}.$$

Check if these vectors are lin. dependent:

$$\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{pmatrix} \begin{matrix} \\ -4 \\ -7 \end{matrix}; \quad \text{reduce}$$

$$\begin{pmatrix}1 & 2 & 3\\0 & -3 & -6\\0 & -6 & -12\end{pmatrix} \qquad \begin{pmatrix}1 & 2 & 3\\0 & 1 & 2\\0 & 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 0\end{pmatrix} \Rightarrow \quad V_3 = -V_1 + 2V_2$$

$\Rightarrow$ linearly dependent.

Then $h(t) = -f(t) + 2g(t)$

$\Rightarrow$ linearly dependent.