Fri., Sep. 12, 2008 **MATH 262** **Quiz 1**

Name: (print) Solutions.

Each problem is worth 2 points. Provide brief explanations for the answers that deserve them in your opinion.

1. Find the rank of the matrices

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ -1 -1

$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

this is rref of the matrix.

rank = 1.

(b) $\begin{pmatrix} 7 & 1 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{7} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

This is a particular case of Exercise 46 in 1.3.

rank = 3.

2. Write the system

$$\begin{cases} x & +2y & +3z & = 1 \\ 4x & +5y & +6z & = 4 \\ 7x & +8y & +9z & = 9 \end{cases}$$

in the matrix form $Ax = b$.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$$

Please turn over...

3. Compute the products if they are defined

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \end{pmatrix}$

not defined;

3 columns in the matrix,

2 elements in the vector

(b) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} + 0 \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 0 \cdot \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

cf Exercise 36 in 1.3

4. Let $x = \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix}$ and $y = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. Find a matrix A of rank one such that $Ax = y$.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

Hint: 1) Compute the product in terms of rows.

2) Any matrix of the form

$$\begin{pmatrix} a & b & c \\ ka & kb & kc \\ ma & mb & mc \end{pmatrix}, \quad a \neq 0$$

has rank 1.

Cf Exercise 30 in 1.3.