Solutions

Each problem is worth 2 points. Provide brief explanations for the answers that deserve them in your opinion.

1. Find the rank of the matrices
   \[
   \begin{pmatrix}
   1 & 1 & 1 \\
   1 & 1 & 1 \\
   1 & 1 & 1
   \end{pmatrix}
   \begin{pmatrix}
   7 & 1 & -1 \\
   0 & 4 & 3 \\
   0 & 0 & 1
   \end{pmatrix}
   \]
   \[
   \begin{pmatrix}
   1 & \frac{1}{7} & -\frac{1}{7} \\
   0 & 1 & \frac{3}{7} \\
   0 & 0 & 1
   \end{pmatrix}
   \]
   \[
   \begin{pmatrix}
   1 & \frac{1}{7} & 0 \\
   0 & 1 & 0 \\
   0 & 0 & 1
   \end{pmatrix}
   \]
   This is a particular case of Exercise 46 in 1.3.
   
   rank = 3.

2. Write the system
   \[
   \begin{align*}
   x + 2y + 3z &= 1 \\
   4x + 5y + 6z &= 4 \\
   7x + 8y + 9z &= 9
   \end{align*}
   \]
   in the matrix form \( Ax = b \).
   \[
   \begin{pmatrix}
   1 & 2 & 3 \\
   4 & 5 & 6 \\
   7 & 8 & 9
   \end{pmatrix}
   \begin{pmatrix}
   x \\
   y \\
   z
   \end{pmatrix}
   =
   \begin{pmatrix}
   1 \\
   4 \\
   9
   \end{pmatrix}
   \]

Please turn over...
3. Compute the products if they are defined

(a) \[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{pmatrix}
\begin{pmatrix}
7 \\
8
\end{pmatrix}
\]

(b) \[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{pmatrix}
\begin{pmatrix}
1 \\
0
\end{pmatrix}
= 1 \begin{pmatrix} 4 \\ 4 \end{pmatrix} + 0 \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 0 \begin{pmatrix} 3 \\ 6 \end{pmatrix}
= \begin{pmatrix} 4 \\ 4 \end{pmatrix}
\]

not defined;
3 columns in the matrix,
2 elements in the vector

4. Let \( x = \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} \) and \( y = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \). Find a matrix \( A \) of rank one such that \( Ax = y \).

\[
\begin{pmatrix}
1 & 2 & 1 \\
0 & 0 & 0 \\
\frac{1}{2} & 1 & \frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
5 \\
3 \\
-9
\end{pmatrix}
= \begin{pmatrix}
2 \\
0 \\
1
\end{pmatrix}
\]

Hint: 1) Compute the product in terms of rows.
2) Any matrix of the form

\[
\begin{pmatrix}
a & b & c \\
ka & kb & kc \\
ma & mb & mc
\end{pmatrix}
\]

has rank 1.

\[\text{Cf Exercise 36 in 1.3}\]