Wed, Oct 8, 2008 **MATH 262** **Midterm Test 1**

Name: (print) Solutions

CSUN ID No. :

This test includes 7 questions, on 7 pages. The duration of the test is 50 minutes.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	total

Note: Write solutions in the space provided. If you run out of space you may use the back side of the page. Show all your steps. The test is closed books/notes. Graphing calculators are not allowed.

1. (5 points) Find the reduced row-echelon form for the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} +1$$

$$rref(A) = \begin{pmatrix} 1 & 0 & 3 & 2 & 3 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Note: The pivot columns in rref(A) (those that have leading 1's) must have all other elements = 0

2. (5 points) Solve the system of linear equations (you may use the answer to problem 1)

$$\begin{cases} \quad\quad x_2 + x_3 + x_4 &= 1 \\ x_1 - x_2 + 2x_3 + x_4 &= 2 \end{cases}$$

Augmented matrix:

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 1 & 2 \end{array}\right)$$

$$\text{rref} = \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 & \\ \multicolumn{5}{c}{\left(\begin{array}{cccc|c} 1 & 0 & 3 & 2 & 3 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right)} \\ \text{pivot} & \text{pivot} & \text{free var} & \text{free var} & \end{array}$$

(Using part (a))

$x_3 = s$

$x_4 = t$

$x_1 = 3 - 3s - 2t$

$x_2 = 1 - s - t$

In vector form,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 - 3s - 2t \\ 1 - s - t \\ s \\ t \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} -3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

3. (9 points) Find the rank of the matrices

(a) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$ (c) $\begin{pmatrix} 7 & 17 & 8 \\ 0 & 9 & 33 \\ 0 & 0 & 11 \end{pmatrix}$

(a) rank = 4

Rearrange the rows to obtain

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) rank = 2

$$\begin{pmatrix} 1 & 4 & 7 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 1 & 4 & 7 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(c) rank = 3

upper triangular, nonzero entries on the diagonal!

4. (10 points) Consider the matrices

$$A = \begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix} \qquad C = \begin{pmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad F = \begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$$

Classify the matrices by type of the linear transformation that they represent and fill in the blanks in the following sentences.

Matrix __B__ represents a scaling

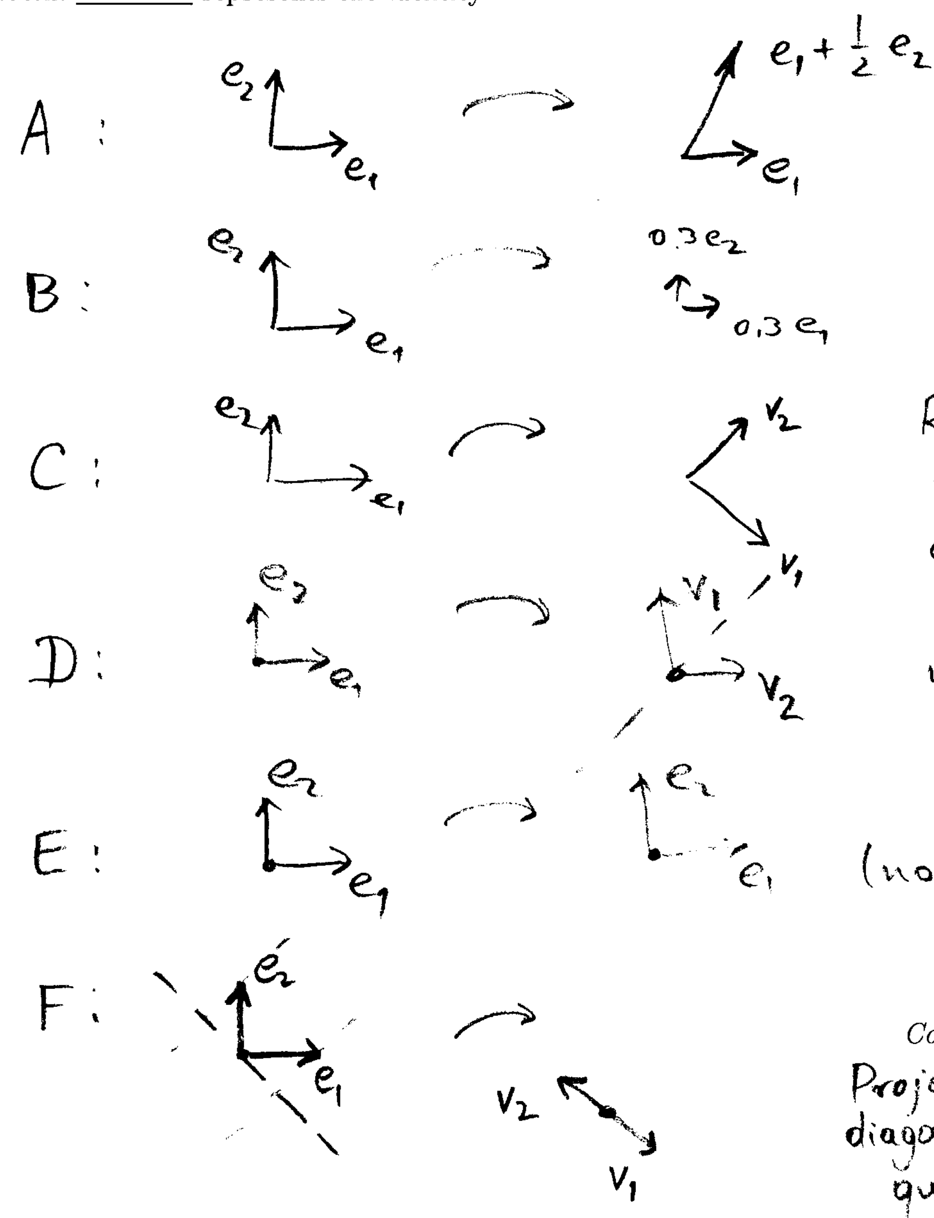
Matrix __C__ represents a rotation

Matrix __F__ represents a projection

Matrix __D__ represents a reflection

Matrix __A__ represents a shear

Matrix __E__ represents the identity



Rotation by arcsin(0.8) clockwise

Reflection with resp. to the diagonal of 1st quadrant

(no change)

Continued...

Projection onto the diagonal of 2nd-4th quadrant.

5. (10 points) Are the following transformations linear? For those that are linear find their matrices.

(a) $\begin{cases} y_1 = x_1 x_2 \\ y_2 = x_1 \end{cases}$ (b) $\begin{cases} y_1 = x_3 \\ y_2 = 2(x_1 - x_2) \\ y_3 = x_3 - 3(x_1 - x_2) \end{cases}$ (c) $\begin{cases} y_1 = x_1 + x_3 - 1 \\ y_2 = 2x_1 + x_2 - 2 \\ y_3 = 3x_1 - x_3 - 3 \end{cases}$

Not linear because of $x_1 x_2$

Check

$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ y = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ y = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Linear, matrix given below

Not linear because of constant terms $-1, -2, -3$

Check:

$x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \ y = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$

$x = 5 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \ y = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} \neq 5 \cdot \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 2 & -2 & 0 \\ -3 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Continued...

6. (10 points) (a) Find a nonzero 2×2 matrix such that $A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Examples: $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. .

From the general form,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & d^2 + bc \end{pmatrix}$$

simlest is $a = 0$, $d = 0$, $bc = 0$, but $b \neq 0$ or $c \neq 0$.

Also, $A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$ . . .

(b) Find a 2×2 matrix not equal to the identity, such that $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Any reflection matrix

Examples: $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$A = \begin{pmatrix} 2u_1^2 - 1 & 2u_1u_2 \\ 2u_1u_2 & 2u_2^2 - 1 \end{pmatrix}$

where $u_1^2 + u_2^2 = 1$. .

7. (10 points) State which of the following statements are true or false.

(a) The matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is in reduced row-echelon form (rref).

(b) A system of four equations with three unknowns is *always* inconsistent.

(c) If A is a 3×4 matrix (3 rows, 4 columns) and B is a 4×5 matrix (4 rows, 5 columns) then AB will be a 5×3 matrix.

(d) If a 2×2 matrix has all non-zero entries then it is invertible.

(e) There is a 3×4 matrix of rank 4.

(f) The inverse of an 3×4 matrix is a 4×3 matrix

(g) The equation $A^2 = A$ holds for all 2×2 matrices representing a projection.

Answers:

(a) true

(b) false; Example $x_1 = 1$, repeated 4 times

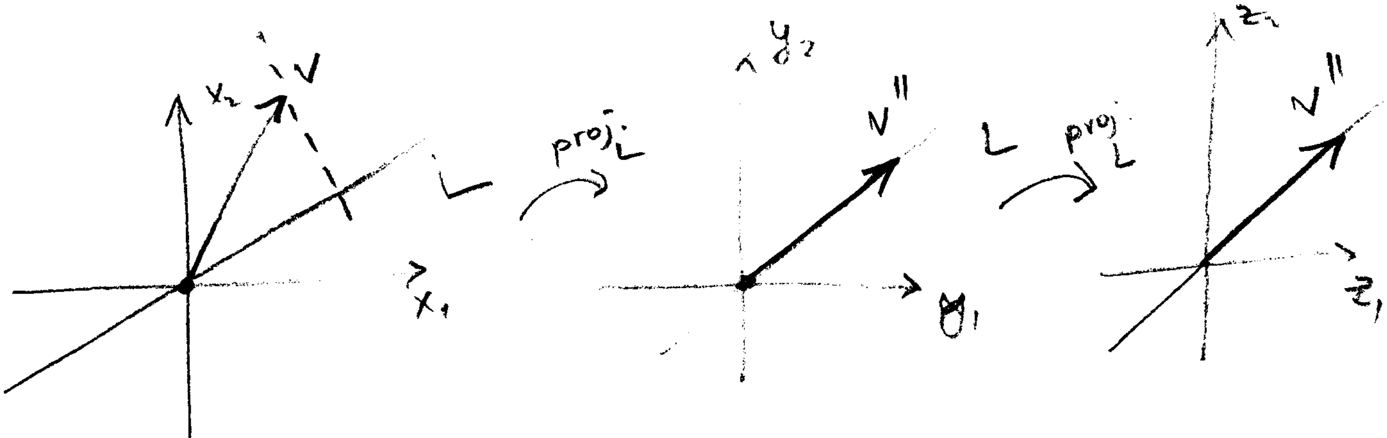
(c) false; it is 3×5

(d) false; example $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(e) false, rank ≤ 3.

(f) false; inverse only works for square matrices

(g) true; illustration:



(After we apply projection once, all vectors on L remain unchanged.)