Solving systems of linear equations

Write the augmented matrix of the system. Place a cursor in the top entry of the first nonzero column of this matrix.

Step 1 If the cursor entry is zero, swap the cursor row with some row below to make the cursor entry nonzero.

Step 2 Divide the cursor row by the cursor entry.

Step 3 Eliminate all other entries in the cursor column, by subtracting suitable multiples of the cursor row from the other rows.

Step 4 Move the cursor one row down and one column to the right. If the new cursor entry and all entries below are zero, move the cursor to the next column (remaining in the same row). Repeat the last step if necessary.

Return to Step 1.

The process ends when we run out of rows or columns. Then, the matrix is in reduced row-echelon form (rref).

Write down the linear system corresponding to this matrix, and solve each equation in the system for the leading variable. You may choose the nonleading variables freely; the leading variables are then determined by these choices. If the echelon form contains the equation $0 = 1$, then there are no solutions; the system is *inconsistent*.

The operations performed in Steps 1, 2, and 3 are called *elementary row operations*: Swap two rows, divide a row by a scalar, or subtract a multiple of a row from another row.

Reduced row-echelon form

A matrix is in *reduced row-echelon form* if it satisfies all of the following conditions:

a. If a row has nonzero entries, then the first nonzero entry is 1, called the *leading* 1 in this row.[10]
b. If a column contains a leading 1, then all other entries in that column are zero.
c. If a row contains a leading 1, then each row above contains a leading 1 further to the left.