MATH 262   FINAL   Spring 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID
number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must
work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX
IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive
no credit, while an incorrect answer accompanied by some correct work may receive partial
credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Find an orthonormal basis of the kernel of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

2. (30 points) Consider the linear system

$$\begin{cases} x + y - z & = 2 \\ x + 2y + z & = 3 \\ x + y + (k^2 - 5)z & = k \end{cases}$$

   where k is an arbitrary constant.

   a. For which value(s) of k this system is *inconsistent*?

   b. For which value(s) of k does this system have one solution? Find the solution.

   c. For which value(s) of k does this system have infinitely many solutions? Find all
      the solutions.

3. (30 points) Find the QR factorization of the matrix

$$A = \begin{bmatrix} 4 & 25 \\ 0 & 0 \\ 3 & -25 \end{bmatrix}$$

4. (40 points) Consider the transformation $T(f(t)) = t(f'(t))$ from P_2 to P_2.

   a. Show that the transformation T is linear.

   b. Find the kernel and the nullity of the transformation T.

   c. Use part (b) to find the rank of the transformation T.

   d. Is the transformation T an isomorphism?

HEY, THERE'S MORE—TURN THE PAGE OVER!

5. (20 points) Consider the matrix

$$A = \begin{bmatrix} 1 & k \\ 1 & 1 \end{bmatrix}$$

where k is an arbitrary constant.

    a. For which values of k does the matrix A have two distinct real eigenvalues?

    b. For which values of k does the matrix A have no real eigenvalue?

6. (30 points) Consider a linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^2$. We are told that the matrix of T with respect to the basis $\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ is $\begin{bmatrix} 1 & 9 \\ 9 & 7 \end{bmatrix}$.

Find the standard matrix of T.

7. (30 points) Consider two distinct numbers, a and b. We define the function

$$f(t) = det \begin{bmatrix} 1 & 1 & 1 \\ a & b & t \\ a^2 & b^2 & t^2 \end{bmatrix}$$

    a. Show that $f(t)$ is a quadratic function. What is the coefficient of t^2?

    b. Explain why $f(a) = f(b) = 0$. Conclude that $f(t) = k(t-a)(t-b)$, for some constant k. Find k, using your work in part (a).

    c. For which values of t is the matrix is invertible?