ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Consider V the subset of P_2 defined by

$$V = \left\{ p(t) : \int_0^1 p(t)\, dt = 0 \right\}$$

   a. Show that V is a subspace of P_2.

   b. Find a basis for V.

2. (30 points) Let V be the set of 3×3 matrices A such that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in the kernel of A. Is V a subspace of $\mathbf{R}^{3\times 3}$?

3. (20 points) Find the QR factorization of the matrix

$$A = \begin{bmatrix} 4 & 25 \\ 0 & 0 \\ 3 & -25 \end{bmatrix}$$

4. (30 points) Consider the linear system

$$\begin{cases} x + y - z & = 2 \\ x + 2y + z & = 3 \\ x + y + (k^2 - 5)z & = k \end{cases}$$

   where k is an arbitrary constants.

   a. For which value(s) of k does this system have a unique solution? Find the solution.

   b. For which value(s) of k does this system have infinitely many solutions? Find all the solutions.

   c. For which value(s) of k is the system *inconsistent*?

HEY, THERE'S MORE—TURN THE PAGE OVER!

5. (20 points) Consider the transformation $T(f(t)) = t(f'(t))$ from P_2 to P_2.

   a. Show that the transformation T is linear.

   b. Find the kernel and the nullity of the transformation T.

   c. Use part (b) to find the rank of the transformation T.

   d. Is the transformation T an isomorphism?

6. (20 points) Consider the matrix

$$A = \begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$$

   a. Find all real eigenvalues of A with their algebraic multiplicities.

   b. Find an eigenvector corresponding to the eigenvalue $\lambda = 1$

7. (40 points) Consider a linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^2$. We are told that the matrix of T with respect to the basis $\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ is $\begin{bmatrix} 1 & 9 \\ 9 & 7 \end{bmatrix}$.

   Find the standard matrix of T.

8. (20 points) Consider two distinct numbers, a and b. We define the function

$$f(t) = det \begin{bmatrix} 1 & 1 & 1 \\ a & b & t \\ a^2 & b^2 & t^2 \end{bmatrix}$$

   a. Show that $f(t)$ is a quadratic function. What is the coefficient of t^2?

   b. Explain why $f(a) = f(b) = 0$. Conclude that $f(t) = k(t-a)(t-b)$, for some constant k. Find k, using your work in part (a).

   c. For which values of t is the matrix is invertible?