

1. Emile and Gertrude are brother and sister. Emile has twice as many sisters as brothers, and Gertrude has just as many brothers as sisters. How many children are there in this family?

2. Consider the linear system

$$\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (k^2 - 5)z = k \end{cases}$$

where k is a constant. For which value(s) of k does this system have a unique solution? Infinitely many solutions? For which value(s) of k is the system inconsistent?

3. Find a system of linear equations with three unknowns whose solutions are the points on the line through $(1, 1, 1)$ and $(3, 5, 0)$.

4. Prof. Bretscher has 32 bills in his wallet, in the denominations of \$1, 5, and 10, worth \$100 in total. How many does he have of each denomination?

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For the following systems write augmented matrices and transform them to reduced row-echelon form using the algorithm of Gauss-Jordan elimination. Find the solutions of the linear systems (if they exist).

$$5. \begin{cases} x + y = 1 \\ 2x - y = 5 \\ 3x + 4y = 2 \end{cases}$$

$$6. \begin{cases} x_1 - 7x_2 + x_5 = 3 \\ x_3 - 2x_5 = 2 \\ x_4 + x_5 = 1 \end{cases}$$

$$7. \begin{cases} x_4 + 2x_5 - x_6 = 2 \\ x_1 + 2x_2 + x_5 - x_6 = 0 \\ x_1 + 2x_2 + 2x_3 - x_5 + x_6 = 2 \end{cases}$$

8. Determine which of the matrices below are in reduced row-echelon form:

$$(a) \begin{pmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(b) \begin{pmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(c) \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$(d) (0 \ 1 \ 2 \ 3 \ 4)$$