1. (a) Let u, v be vectors in $\mathbb{R}^3$. Show that

$$u \times v = -v \times u$$

(the cross product is skew-symmetric).

(b) Show that for any $u \in \mathbb{R}^3$, $u \times u = 0$.

2. Define the *mixed product* of three vectors $u, v, w \in \mathbb{R}^3$ as

$$[u, v, w] = u \cdot (v \times w).$$

(a) Show that if any two of the vectors u, v, w are equal then $[u, v, w] = 0$. [Hint: Use the previous problem and the fact that $v \times w$ is perpendicular to both v and w.]

(b) Show that

$$\begin{aligned} [u, v, w] &= -[v, u, w] \\ &= -[u, w, v] \\ &= -[w, v, u] \\ &= [v, w, u] \\ &= [w, u, v] \end{aligned}$$

(the pattern is that an odd permutation of u, v, w makes the product change sign, while an even permutation preserves it).

[Hint: For the first of the identities use part (a)]