Wed, Feb 12, 2020 **MATH 262** **Quiz 1**

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. If $r \in \mathbb{R}$ and $0 \in V$ is the zero vector, prove that $r0 = 0$. Give a proof based on the Axioms of Vector Space. Show all steps.

WTS: for any vector $w \in V$

$w + r0 = 0$

If $r \neq 0$, $w + r \cdot 0 = 1 \cdot w + r \cdot 0$

$= \frac{1}{r} \cdot rw + r \cdot 0$

$= r\left(\frac{1}{r}w + 0\right) = r\frac{1}{r}w = w$

If $r = 0$, $w + r \cdot 0 = (w + 0) + r \cdot 0$

$= w + (0 + r \cdot 0) = w + (1 \cdot 0 + r \cdot 0)$

$= w + (1 + 0) \cdot 0 = w + 1 \cdot 0$

$= w + 0 = w$

2. If $r \in \mathbb{R}$, $v \in V$, $r \neq 0$ and $rv = 0$ prove that $v = 0$. Show all steps.

$rv = 0$

(A2) $\frac{1}{r} \cdot (rv) = \frac{1}{r} \cdot 0$

(A9, part 1) $\left(\frac{1}{r} \cdot r\right)v = 0$

$1 \cdot v = 0$

(A10) $v = 0$.

Please turn over...

3. Determine whether the following subset of $\mathbb{R}^2$ is a subspace or not:

$$H = \{(v_1, v_2) \in \mathbb{R}^2 : v_1 \geq 0\}.$$

Give a proof.

Not a subspace:

$(1,0) \in H$

but $(-1)\cdot(1,0) = (-1,0) \notin H$

so H is not closed under mult. by scalar.

4. Determine whether the set $S = \{f \in \mathbb{D}(\mathbb{R}) : f'(x) + f(x) = 0, \text{ for all } x \in \mathbb{R}\}$ is a subspace of $\mathbb{D}(\mathbb{R})$ or not. Give a proof.

A subspace:

$f_0(x) = 0$ for all $x \in \mathbb{R}$

$\Rightarrow f_0'(x) = 0 \Rightarrow f_0' + f_0 = 0$

$f_0 \in S \Rightarrow S \neq \emptyset.$

If $f, g \in S$ then $\left.\begin{array}{l} f' + f = 0 \\ g' + g = 0 \end{array}\right\} = (f+g)' + (f+g) = 0$

If $r \in \mathbb{R}$, $f \in S$ then $f' + f = 0$

$\Rightarrow rf' + rf = 0$

S is closed for "+" and "$\cdot$"

$\Rightarrow$ subspace.