EXERCISES 2.1

#1: (a)
$$\begin{cases} x + 2y = 7 \\ y = 2 \end{cases} \quad -2R_2 + R_1 \to R_1 \quad \begin{cases} x = 3 \\ y = 2 \end{cases}$$

(c)
$$\begin{cases} x - 3y = 2 \\ y = -1 \\ z = 5 \end{cases} \quad 3R_2 + R_1 \to R_1 : \quad \begin{cases} x = -1 \\ y = -1 \\ z = 5 \end{cases}$$

#2: (a)
$$\begin{cases} x + 2y = 2 \\ z = 3 \end{cases} \quad y\text{ - free} \quad \begin{cases} x = 2 - 2t \\ y = t \\ z = 3 \end{cases}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

(d)
$$\begin{cases} x - 2y + 4z = 1 \\ y - 2z = -1 \end{cases} \quad 2R_2 + R_1 \to R_1 \begin{cases} x = -1 \\ y - 2z = -1 \end{cases}$$

z - free
$$x = -1$$
$$y = -1 + 2t$$
$$z = t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

#3. (a) $\begin{cases} w + 2x \quad + 3z = 1 \\ \quad y - z = 2 \end{cases}$

$x = s$ - free $\quad z = t$ - free

$$\begin{cases} w = 1 - 2s - 3t \\ x = s \\ y = 2 + t \\ z = t \end{cases} ; \quad \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

(c) $\begin{cases} x_1 - x_2 + x_3 - x_4 = 1 \\ x_2 - 2x_3 - x_4 = 3 \\ 2x_2 - 4x_3 - 2x_4 = 6 \end{cases} \quad -2R_2 + R_3 \to R_3$

$\begin{cases} x_1 - x_2 + x_3 - x_4 = 1 \\ x_2 - 2x_3 - x_4 = 3 \\ 0 = 0 \end{cases} \quad R_2 + R_1 \to R_1$

$\begin{cases} x_1 \quad - x_3 - 2x_4 = 4 \\ x_2 - 2x_3 - x_4 = 3 \end{cases} \quad \begin{matrix} x_3 = s \text{ -free} \\ x_4 = t \text{ -free} \end{matrix}$

$$\begin{cases} x_1 = 4 + s + 2t \\ x_2 = 3 + 2s + t \\ x_3 = s \\ x_4 = t \end{cases} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

MATH 262 EXERCISES 2.2

#2. 1. $R_i \leftrightarrow R_j$ inverse: $R_i \leftrightarrow R_j$ (interchange the rows again)

2. $c \neq 0$, $cR_i \to R_i$ inverse: $\frac{1}{c}R_i \to R_i$

3. $cR_i + R_j \to R_j$ inverse: $-cR_i + R_j \to R_j$ (subtract the c-multiple of R_i from the new R_j to obtain the original R_j.)

#5 (a)
$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -3 & 6 \\ -1 & 2 & -2 \end{bmatrix} \xrightarrow{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 0 \\ -1 & 2 & -2 \end{bmatrix}$$

$$\xrightarrow{R_1 + R_3 \to R_3} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[2R_2 + R_1 \to R_1]{-3R_3 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 1 & 0 & -2 & 1 \\ 3 & -1 & -7 & 0 \\ 2 & -3 & -7 & -7 \end{bmatrix} \xrightarrow{-3R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & -1 & -1 & -3 \\ 2 & -3 & -7 & -7 \end{bmatrix}$$

$$\xrightarrow{-2R_1 + R_3 \to R_3} \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & -3 & -3 & -9 \end{bmatrix} \xrightarrow{-R_2 \to R_2} \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & -3 & -3 & -9 \end{bmatrix}$$

$$\xrightarrow{3R_2+R_3\to R_3} \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ RREF.

(d) $$\begin{bmatrix} 1 & -2 & 2 & 11 \\ -1 & 2 & 3 & -1 \\ -2 & 4 & 0 & -14 \end{bmatrix} \xrightarrow{R_1+R_2\to R_2} \begin{bmatrix} 1 & -2 & 2 & 11 \\ 0 & 0 & 5 & 10 \\ -2 & 4 & 0 & -14 \end{bmatrix}$$

$$\xrightarrow{2R_1+R_3\to R_3} \begin{bmatrix} 1 & -2 & 2 & 11 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & 4 & 8 \end{bmatrix} \xrightarrow{\frac{1}{5}R_2\to R_2} \begin{bmatrix} 1 & -2 & 2 & 11 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 8 \end{bmatrix}$$

$$\xrightarrow{-4R_2+R_3\to R_3} \begin{bmatrix} 1 & -2 & 2 & 11 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-2R_2+R_1\to R_1} \begin{bmatrix} 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ RREF

#6(a) $$\begin{bmatrix} 2 & -4 \\ -3 & 6 \\ 1 & 2 \\ -2 & 4 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1\to R_1} \begin{bmatrix} 1 & -2 \\ -3 & 6 \\ 1 & 2 \\ -2 & 4 \end{bmatrix} \xrightarrow{3R_1+R_2\to R_2} \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 1 & 2 \\ -2 & 4 \end{bmatrix}$$

$$\xrightarrow{-R_1+R_3\to R_3} \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 4 \\ -2 & 4 \end{bmatrix} \xrightarrow{2R_1+R_4\to R_4} \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 4 \\ 0 & 0 \end{bmatrix} \xrightarrow{R_2\leftrightarrow R_3} \begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{4}R_2\to R_2} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{2R_2+R_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$ RREF

#6 (b)

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_1} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ RREF}$$

#8. Example in 6 (a)

$$\begin{bmatrix} 2 & -4 \\ -3 & 6 \\ 1 & 2 \\ -2 & 4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 \\ -3 & 6 \\ 2 & -4 \\ -2 & 4 \end{bmatrix}$$

$$\xrightarrow{3R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 \\ 0 & 12 \\ 2 & -4 \\ -2 & 4 \end{bmatrix} \xrightarrow{-2R_1 + R_3 \to R_3} \begin{bmatrix} 1 & 2 \\ 0 & 12 \\ 0 & -8 \\ -2 & 4 \end{bmatrix} \xrightarrow{2R_1 + R_4 \to R_4} \begin{bmatrix} 1 & 2 \\ 0 & 12 \\ 0 & -8 \\ 0 & 8 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{12}R_2 \to R_2} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & -8 \\ 0 & 8 \end{bmatrix} \xrightarrow{8R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 8 \end{bmatrix} \xrightarrow{-8R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

REF

Row interchange in the beginning will generally produce a different REF.

Notice that RREF is still the same:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$