# Math 262 Problems
# Textbook: Robert Messer

1. Let $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 5 \\ 4 & 15 & 6 \end{bmatrix}$

   a. Compute $A^{-1}$

   b. Use to $A^{-1}$ solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$.

2. Let $C[-1,1]$ be the vector space of all real-valued functions that are continuous on the interval $[-1,1]$.

   a. Let $W = \{ f \in C[-1,1] \mid f(-x) = -f(x), \text{ for all } x \in [-1,1] \}$. Prove $W$ is a subspace of $C[-1,1]$.

   b. Let $W = \{ f \in C[-1,1] \mid f(-1) = 0 \text{ or } f(1) = 0 \}$. Give a counterexample to show that $W$ is not a subspace of $C[-1,1]$.

3. Classify each of the following statements as ***always true***, ***sometimes true***, or ***never true***. No justification required.

   a. If $\dim(V) = 3$, and $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are vectors in $V$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a linearly independent set.
   b. If $\dim(V) = 3$, and $\mathbf{v}_1, \mathbf{v}_2$ are vectors in $V$, then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set.
   c. If $\dim(V) = 3$, and $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are vectors in $V$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ spans $V$.
   d. If $\dim(V) = 3$, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in $V$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ spans $V$.
   e. If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $V$, then $\{3\mathbf{v}_1, \frac{1}{2}\mathbf{v}_2, -2\mathbf{v}_3\}$ is also a basis for $V$.
   f. If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $V$, then $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3\}$ is a basis for $V$.

4. Let $L$ be a linear transformation from a vector space $V$ into a vector space $W$ where $\ker(L) = \{\mathbf{0}\}$. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent vectors in $V$, prove that $L(\mathbf{v}_1), L(\mathbf{v}_2), L(\mathbf{v}_3)$ are linearly independent vectors in $W$. (Indicate in your proof where you use the fact that $\ker(L) = \{\mathbf{0}\}$.)

5. Let $P_2$ be the vector space of polynomials of degree less than ***or equal to*** 2. If $L: P_2 \to P_2$ is given by $L(p(x)) = p(0)x + p(1)$,

a. Prove that $L$ is a linear transformation.
b. Find a basis for the kernel of $L$.
c. Find a basis for the range of $L$.
d. Is $L$ onto? Explain.

6. Let $A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$

a. Find the eigenvalues of $A$.
b. Find bases for the eigenspaces of $A$.
c. Express $A$ in the form $A = XDX^{-1}$ where $D$ is a diagonal matrix.
d. ***Use part c.*** to compute $A^6$.

7. Let $L: P_1 \to P_3$ be the linear transformation given by $T(p(x)) = (x^2+1)p(x)$.

a. Find the matrix representation of $L$ relative to the ordered bases $B$, $B'$ where $B = \{1, 2x+4\}$ and $B' = \{4, 3x, 2x^2, x^3\}$.
b. Let $q(x) = 2x+10$. Find $[q(x)]_B$, the coordinate vector of $q(x)$ relative to $B$.
c. ***Use your answer from part a.*** to find $[L(q(x))]_{B'}$.
d. ***Use your answer from part c.*** to find $L(q(x))$.

8. (12 pts.) Let $B = \left( \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right)$ and $B' = \left( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right)$ be ordered bases for $R^3$.

a. Find the transition matrix $P$ such that $P[\mathbf{v}]_B = [\mathbf{v}]_{B'}$ for every $\mathbf{v} \in R^3$.

b. If $\mathbf{v} = 2\begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} - 3\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, find $[\mathbf{v}]_{B'}$