The Gauss-Jordan algorithm

The input of the algorithm is an \( m \times n \) matrix (not necessarily square!), which is typically an augmented matrix of a linear system, however the algorithm works for any matrix with numerical entries.

Start with \( i = 1, \ j = 1 \).

1. If \( a_{ij} = 0 \) swap the \( i \)-th row with some other row below to guarantee that \( a_{ij} \neq 0 \).
   The non-zero entry in the \((i, j)\)-position is called a pivot. If all entries in the column are zero, increase \( j \) by 1.

2. Divide the \( i \)-th row by \( a_{ij} \) to make the pivot entry = 1.

3. Eliminate all other entries in the \( j \)-th column by subtracting suitable multiples of the \( i \)-th row from the other rows.

4. Increase \( i \) by 1 and \( j \) by 1 to choose the new pivot element. Return to Step 1.

The algorithm stops after we process the last row or the last column of the matrix.

The output of the Gauss-Jordan algorithm is the matrix in reduced row-echelon form.

Reduced row-echelon form

A matrix is in reduced row-echelon form (RREF) if it satisfies all of the following conditions.

1. If a row has nonzero entries, then the first non-zero entry is 1 called the leading 1 in this row.

2. If a column contains a leading one then all other entries in that column are zero.

3. If a row contains a leading one the each row above contains a leading one further to the left.

The last point implies that if a matrix in rref has any zero rows they must appear as the last rows of the matrix.