Thu, Feb. 14, 2019 **MATH 255A Midterm 1, Version 2** Prof. V. Panferov

Name: (print) Solutions.

CSUN ID No. : ____________________

This test includes 8 questions (50 points in total), on 8 pages. Page 9 is a formula sheet. The duration of the test is 1 hour 15 minutes.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	8	total

Important: One single-sided page of notes is allowed. No electronic devices except an approved model of graphing calculator. All cellphones must be off and put away completely for the duration of the exam. Show all your work.

1. (6 points) (a) A bacterial population in Lab A starts with 1,000 bacteria at time $t = 0$ and doubles every 12.4 hours. Find a formula for the size of the population after t hours.

$P(t) = P_0 b^t$

$P(0) = P_0 = 1000$

$P(12.4) = 2P_0 = P_0 b^{12.4}$

$b^{12.4} = 2$

$b = 2^{1/12.4}$

$P(t) = 1000 \cdot \left(2^{1/12.4}\right)^t$

$= 1000 \cdot 2^{t/12.4}$

$\approx 1000 \cdot (1.0575)^t$

(or $1000 \cdot e^{0.055899t}$)

(b) Another bacterial population, in Lab B, starts with 1,000 bacteria at time $t = 0$ and increases by 6% every hour. Find a formula for the size of the population after t hours.

$P(t) = P_0 (1.06)^t$

$P(0) = 1{,}000 = P_0$

$P(t) = 1{,}000 \cdot (1.06)^t$

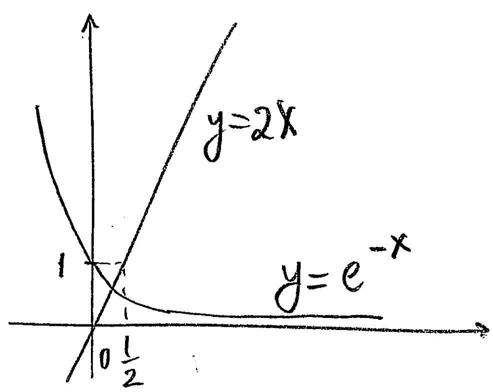
(c) Compare the sizes of the two populations after 24 hours. Which one is growing faster?

Population A: $1{,}000 \cdot (2)^{24/12.4} = 3{,}825.06$

Population B: $1{,}000\,(1.06)^{24} = 4{,}048.93$

The two populations start from the same value when $t=0$, therefore Population B grows faster.

2. (6 points) (a) By graphing, show that the equation $e^{-x} = 2x$ has exactly one solution x.



The solution must be on the interval $(0, \frac{1}{2})$

(b) Using graphing calculator, find the solution of the equation in part (a), accurate to two decimal places.

$\boxed{\text{ZOOM}}$ (ZBox) $\boxed{\text{TRACE}}$

$e^{-x} - 2x$ changes sign between 0.35 and 0.355

$\Rightarrow$ accurate to two decimal places

$x \cong 0.35$

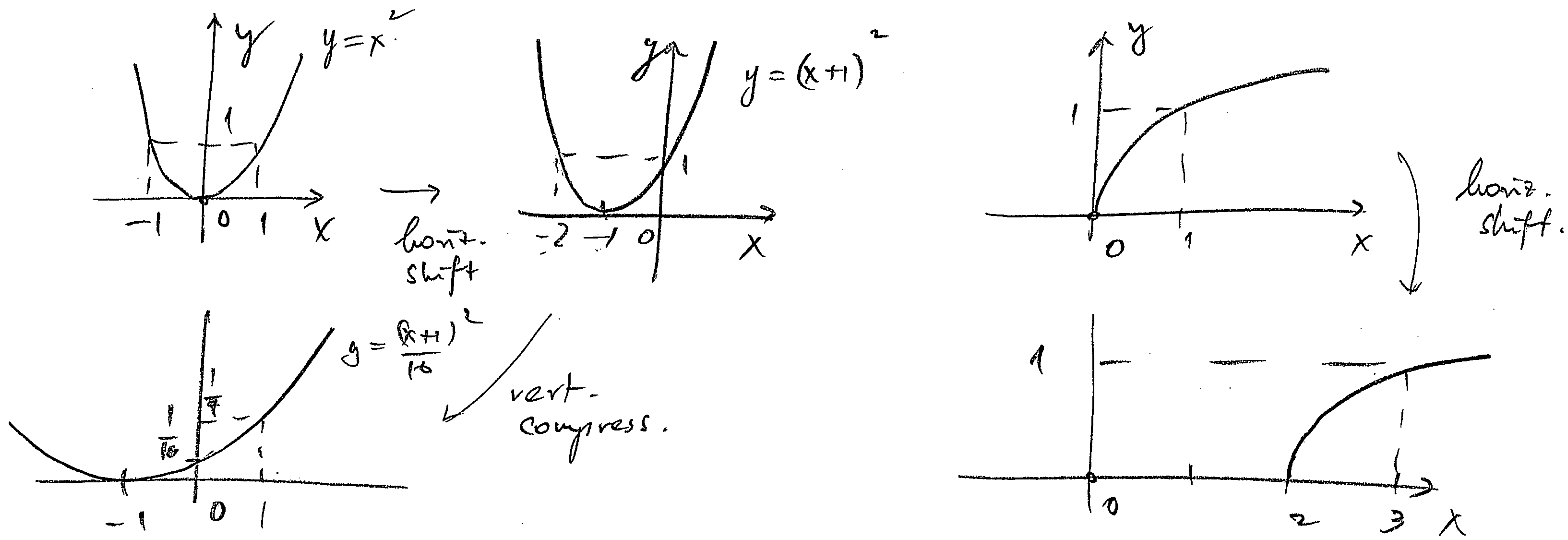
More accurately (using $\boxed{\text{CALC}}$ (zero))

$x \cong 0.357734$

Continued...

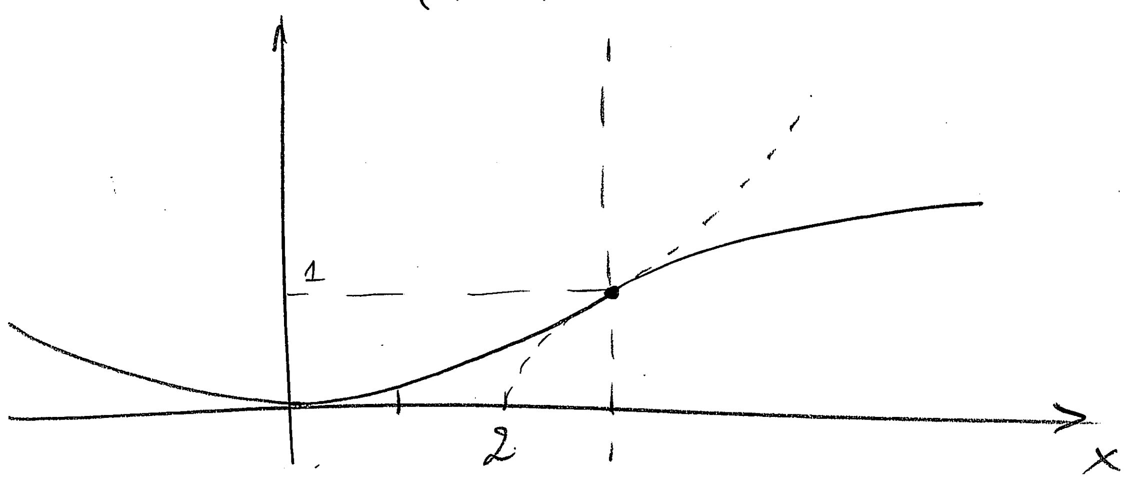
3. (6 points) (a) Using graphing techniques (shifting, reflecting and stretching) sketch the graphs of the functions

$$y = \frac{(x+1)^2}{16} \quad \text{and} \quad y = \sqrt{x-2}.$$



(b) Use the graphs in part (a) to sketch the graph of the following function:

$$y = f(x) = \begin{cases} \frac{(x+1)^2}{16}, & \text{for } x \le 3 \\ \sqrt{x-2}, & \text{for } x > 3\,. \end{cases}$$



(c) Compute the values $f(-1)$, $f(0)$, $f(1)$, $f(3)$ (if defined). State the domain and the range of f.

x	-1	0	1	3
y	0	$\frac{1}{16}$	$\frac{1}{4}$	1

Domain: $\mathbb{R}\ (= (-\infty, \infty))$

Range: $[0, \infty)$

Continued...

4. (6 points) (a) If the variables D and L are related through proportionality $D \propto L^3$, and D increases 100-fold, what happens to L?

$$D \propto L^3 \Rightarrow L \propto D^{1/3}$$

$$\frac{D_2}{D_1} = 100 \Rightarrow \frac{L_2}{L_1} = \frac{\cancel{c}\, D_2^{1/3}}{\cancel{c}\, D_1^{1/3}} = \left(\frac{D_2}{D_1}\right)^{1/3} = 100^{1/3} = \simeq 4.64.$$

L increases by a factor of ≈ 4.64.

(b) Express the following function

$$y = \frac{\sqrt{25x} + 9x^{1/2}}{36x} = \frac{5\sqrt{x} + 9x^{1/2}}{36x}$$

as a proportionality relation $y \propto x^b$.

$$= \frac{5x^{1/2} + 9x^{1/2}}{36x} = \frac{14x^{1/2}}{36x} = \frac{7}{18} x^{-1/2}$$

$$y \propto x^{-1/2}.$$

5. (8 points) (a) Solve the equations:

(i) $e^{5x} = 4e^{2x}$ $\quad \div e^{2x}$

$$\frac{e^{5x}}{e^{2x}} = 4$$

$$e^{3x} = 4$$

$$3x = \ln(4)$$

$$x = \frac{\ln(4)}{3} \approx 0.462$$

(ii) $b^{9.3} = 4$.

$$\left(b^{9.3}\right)^{\frac{1}{9.3}} = 4^{\frac{1}{9.3}}$$

$$b = 4^{\frac{1}{9.3}} \approx 1.1607$$

(b) Write the expression in terms of base e (or ln) and simplify where possible:

$$\log_8(ex - e) - \frac{1}{\ln 8}.$$

$$\frac{\ln(e(x-1))}{\ln 8} - \frac{1}{\ln 8}$$

$$\frac{\ln e + \ln(x-1)}{\ln 8} - \frac{1}{\ln 8} = \frac{1 + \ln(x-1) - 1}{\ln 8}$$

$$= \frac{\ln(x-1)}{\ln 8}.$$

(c) Simplify using properties of logarithms (show work without referring to graphing calculator):

$$\ln e - \ln 1 + \ln e^{999}.$$

$$1 - 0 + 999 = 1{,}000.$$

6. (6 points) (a) Find the line of best fit for the data:

x	0	1	2	3
y	2.00	2.71	3.21	3.50

$\boxed{\text{STAT}} \rightarrow \boxed{\text{EDIT}}$; enter data onto L_1, L_2

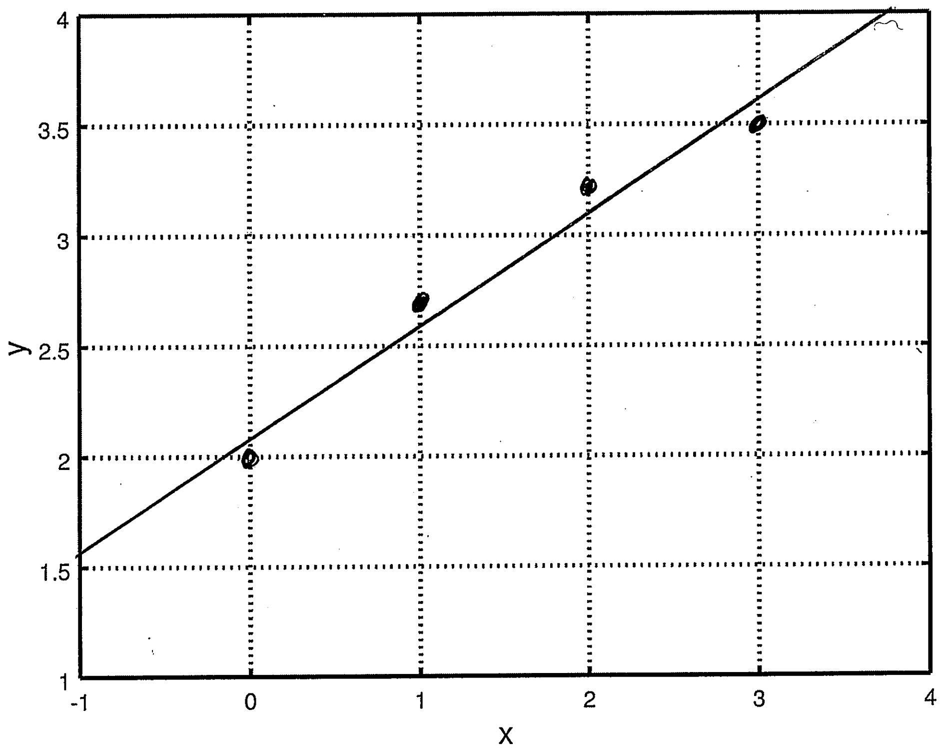
$\boxed{\text{STAT}} \rightarrow \boxed{\text{CALC}}$; LinReg $(ax+b)$ L_1, L_2 ;

Sketch a scatter plot of the data and the best fit line:

$y = ax + b$

$a = 0.5$

$b = 2.105$



(b) Use the equation of the best fit line to obtain a prediction for the y-value when $x = 2.5$.

$x = 2.5$

$y = 0.5 \cdot 2.5 + 2.105$

$= 1.25 + 2.105 = 3.855$

7. (6 points) The tide height, in ft, at a Southern California beach are given in the following table:

Time	Height (ft)	Tide
2:00am	4.3	High
8:00am	1.7	Low
2:00pm	4.3	High

Let

$$T(t) = A\cos(B(t - C)) + D$$

denote the height of the tide t hours after midnight.

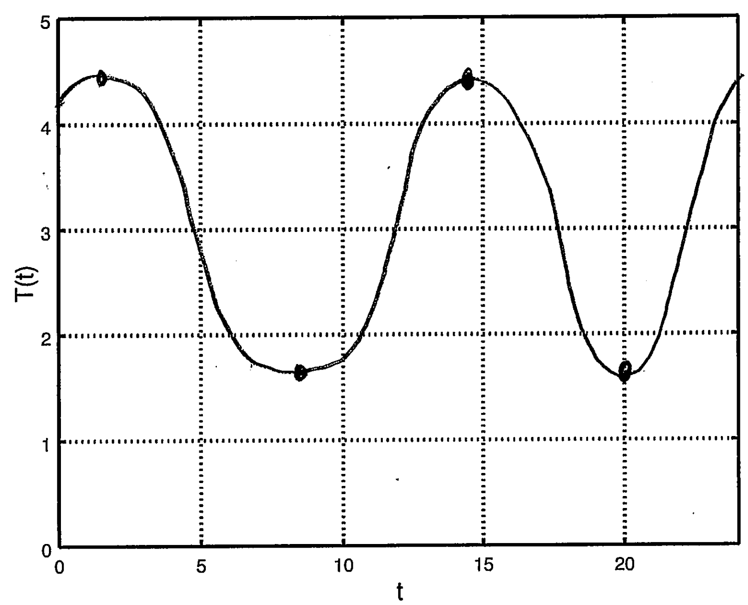
(a) Find values of A, B, C, and D such that the function fits the data.

$A = \frac{y_{max} - y_{min}}{2} = 1.3$ $\qquad B = \frac{2\pi}{\text{period}} = \frac{2\pi}{12} = \frac{\pi}{6}$

$D = \frac{y_{max} + y_{min}}{2} = 3.0$

$C = 2$ (max y-value is 4.3) $\qquad T(t) = 3 + 1.3\cos\left(\frac{\pi}{6}(t-2)\right)$

(b) Sketch a graph of the function over the interval $[0, 24]$.



(c) According to this model, what was the tide height at 10:00am?

$T(10) = 3 + 1.3\cos\left(\frac{\pi}{6}\cdot 8\right) = 2.35$ [ft].

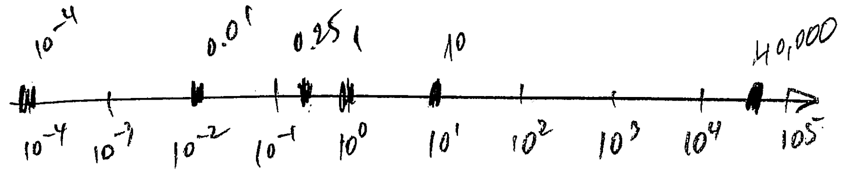
Continued...

8. (6 points) (a) Sketch the indicated points on a logarithmic scale (use base 10):

X	10^{-4}, 0.01, 0.25, 1, 10, 40000
$\log_{10} X$	-4 -2 -0.6 0 1 4.6

Note:

$\log_{10} 0.25 = \log_{10} \frac{1}{4}$
$\simeq -\log_{10} 4$
$\simeq -0.6$



$\log_{10} 40{,}000$
$= \log_{10} 4 + \log_{10} 10^4$
$\simeq 0.6 + 4 = 4.6$

(b) Sketch a log-log graph of the function $y = 1000x^{-1}$ over the interval $[0.001, 1000]$. On the vertical axis, mark the values that correspond to $x = 10^{-3}, 10^{0}, 10^{3}$.

$\log_{10} y = \log_{10} 1000 + \log_{10} x^{-1} = 3 - \log_{10} x$

$Y = 3 - X \qquad (Y = \log_{10} y;\ X = \log_{10} x)$

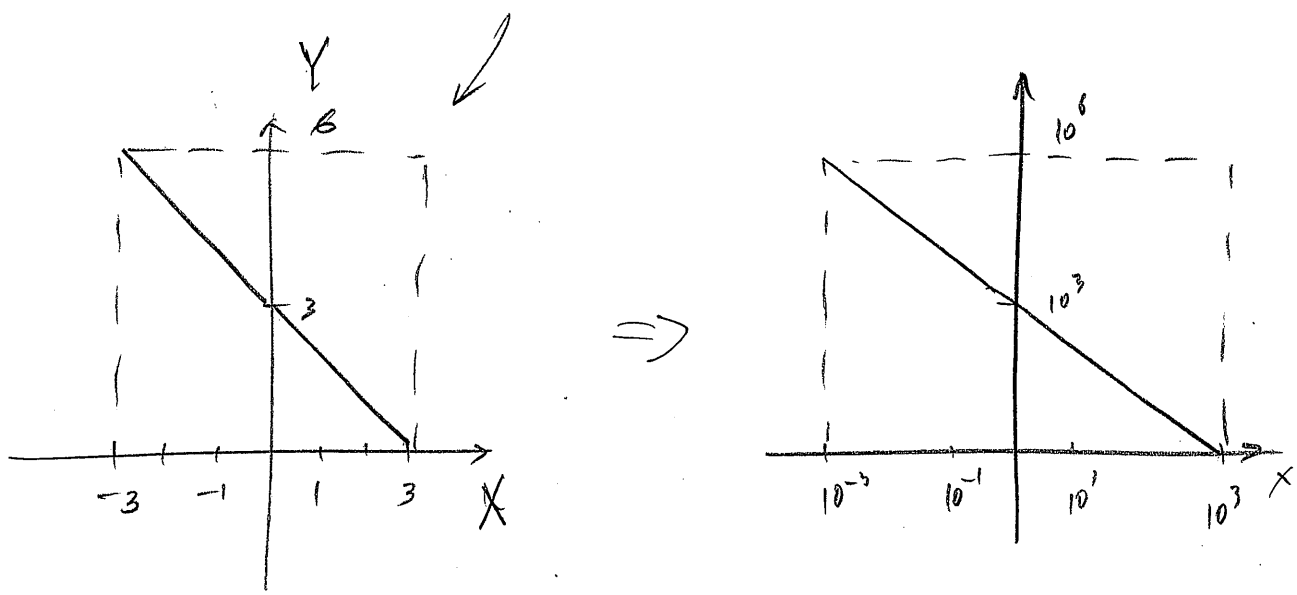


Table of formulas

<u>Linear Regression</u> (line of best fit): $y = ax + b$;   Linreg$(ax+b)$ L_1, L_2

<u>Trigonometric Functions</u> $y = A\cos(B(t-C)) + D$

$A = \frac{y_{\max} - y_{\min}}{2}$ – amplitude; $D = \frac{y_{\max} + y_{\min}}{2}$ – mean value

$B = \frac{2\pi}{\text{period}}$; C – location of maximum.

<u>Proportionality</u> $y \propto x^p$ means $y = cx^p$

$$y \propto x^p \quad \Leftrightarrow \quad x \propto y^{1/p}$$

$$y \propto x^p,\ z \propto y^q \quad \Rightarrow \quad z \propto x^{pq}.$$

<u>Compound Interest</u>

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ – compounded n times per year

$A(t) = Pe^{rt}$ – compounded continuously.

<u>Half-life and doubling time</u>

$$P(t) = P_0\, b^t \quad \Rightarrow \quad t_D = \frac{\ln 2}{\ln b} \quad (b > 1)$$

$$t_{1/2} = \frac{\ln \frac{1}{2}}{\ln b} \quad (b < 1).$$

<u>Laws of exponents</u>

$$x^p x^q = x^{pq}$$

$$\frac{x^p}{x^q} = x^{p-q}; \qquad (xy)^p = x^p y^p$$

$$\left(x^p\right)^q = x^{pq}; \qquad \left(\frac{x}{y}\right)^p = \frac{x^p}{y^p}.$$

<u>Laws of logarithms</u>

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^p = p \log_a x.$$

<u>Change of base</u>

$$\log_a x = \frac{\log_b x}{\log_b x}.$$

The end.