1. (5 points) The concentration of a drug in a patient's bloodstream \( t \) hours after it was injected is given by

\[
A(t) = \frac{0.12t}{t^2 + 4} \quad \text{(in [mg/l]).}
\]

Find and interpret \( \lim_{t \to \infty} A(t) \).

\[
\lim_{t \to \infty} A(t) = \lim_{t \to \infty} \frac{t^2(\frac{0.12}{t})}{t^2(1 + \frac{4}{t^2})} = \lim \frac{0.12}{1 + \frac{4}{t^2}} = 0.
\]

As time goes on, the amount of drug in the bloodstream decreases to zero.
2. (8 points) A particle is moving along a straight line; its position at time \( t \) is given by \( s(t) = 10 - \frac{gt^2}{2} \) (\( g \) is a constant).

(a) Find the average speed of the particle over the interval \( 1 \leq t \leq 2 \).

\[
\text{Average speed} = \frac{\Delta s}{\Delta t}
\]

\[
\Delta s = s(2) - s(1) = \left(10 - \frac{4g}{2}\right) - \left(10 - \frac{g}{2}\right)
\]

\[
= -\frac{3g}{2}
\]

\( \Delta t = 1 \)

\[
\frac{\Delta s}{\Delta t} = -\frac{3g}{2}
\]

(b) Use the definition of the derivative to find the instantaneous speed of the particle at \( t = 1 \).

\[
\text{Definition: } \quad s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}
\]

\[
= \lim_{h \to 0} \frac{(10 - \frac{g(t+h)^2}{2}) - (10 - \frac{gt^2}{2})}{h}
\]

\[
= \lim_{h \to 0} \frac{(-\frac{g}{2}) \frac{(t+h)^2-t^2}{h}}{h}
\]

\[
= (-\frac{g}{2}) \lim_{h \to 0} \frac{t^2 + 2th + h^2 - t^2}{h}
\]

\[
= (-\frac{g}{2}) \lim_{h \to 0} (2t + h) = -\frac{g}{2} (2t) = -gt
\]

At \( t = 1 \) : \( s'(1) = -g \).

Continued...
3. (8 points) For the function \( f(x) = \begin{cases} 0, & x < 0 \\ x(7 - x), & 0 \leq x \leq 7 \\ 7x, & x > 7 \end{cases} \)

find all points where \( f(x) \) is discontinuous. Find the left and the right limits of \( f(x) \) at these points.

Transition points: \( x = 0 \) and \( x = 7 \).

\( x = 0 \):
\[
\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 0 = 0.
\]
\[
\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x(7 - x) = 0. \quad \therefore 0 = 0.
\]

Right and left limits are the same
\( \Rightarrow f(x) \) is continuous at \( x = 0 \).

\( x = 7 \):
\[
\lim_{x \to 7^-} f(x) = \lim_{x \to 7^-} x(7 - x) = 7 \cdot 0 = 0.
\]
\[
\lim_{x \to 7^+} f(x) = \lim_{x \to 7^+} 7x = 7 \cdot 7 = 49.
\]

Right and left limits are different
\( \Rightarrow f(x) \) is discontinuous at \( x = 7 \)

Sketch (optional,)

[Diagram showing the function \( f(x) \) with points and limits indicated.]
4. (8 points) Find the derivatives of the functions

(a) \[ f(x) = \frac{5x + 6}{\sqrt{x}} = \frac{5x}{\sqrt{x}} + \frac{6}{\sqrt{x}} = 5\sqrt{x} + \frac{6}{\sqrt{x}} = 5x^\frac{1}{2} + 6x^{-\frac{1}{2}} \]

\[ f'(x) = 5 \cdot \frac{1}{2} \cdot x^\frac{1}{2} - \frac{1}{2} + 6 \cdot \left(-\frac{1}{2}\right) \cdot x^{-\frac{3}{2}} \]

\[ = \frac{5}{2} x^{-\frac{1}{2}} - 3 x^{-\frac{3}{2}} \]

\[ = \frac{5}{2 \sqrt{x}} - \frac{3}{x \sqrt{x}}. \]

(b) \[ g(t) = (e^{-t} + 2t)^2 \]

\[ g'(t) = 2(e^{-t} + 2t)(-e^{-t} + 2) \]

\[ \text{Chain rule: } f(u) = u^2, \ f'(u) = 2u \]

\[ u = h(t) = e^{-t} + 2t, \ h'(t) = -e^{-t} + 2 \]

\[ g'(t) = f'(u)h'(t) = 2(e^{-t} + 2t)(-e^{-t} + 2). \]
5. (8 points) Find the limits, if they exist. If they don’t explain why.

(a) \( \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \) 

\[
\frac{\sqrt{x} - 4}{x - 16} = \frac{(\sqrt{x} - 4)(\sqrt{x} + 4)}{(x - 16)(\sqrt{x} + 4)} = \frac{x - 16}{(x - 16)(\sqrt{x} + 4)} = \frac{1}{\sqrt{x} + 4} (x \neq 16)
\]

\[
\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} = \frac{1}{\sqrt{16} + 4} = \frac{1}{8}.
\]

(b) \( \lim_{x \to 0} \frac{|x|}{x} \) 

\[
\frac{|x|}{x} = \begin{cases} 
\frac{x}{x}, & x > 0 \\
\frac{-x}{x}, & x < 0 
\end{cases} = \begin{cases} 
1, & x > 0 \\
-1, & x < 0 
\end{cases}
\]

\[
\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} -1 = -1
\]

\[
\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} 1 = 1
\]

Right and left limits are different \( \Rightarrow \) the limit \( x \to 0 \) Does Not Exist.
6. (6 points) In this problem you are given the graphs of six functions. Under each graph indicate for which values of \( x \) the function is (a) discontinuous (b) not differentiable. (You do not need to explain why.)

1. (a) none
   (b) \( x = 0 \)

2. (a) none
   (b) \( x = -2, -1, 0, 1, 2 \)

3. (a) none
   (b) none

4. (a) \( x = -1 \)
   (b) \( x = -1 \)

5. (a) \(-1 \leq x \leq 1\)
   (b) \(1 \leq x \leq 1\)

6. (a) \( x = 1 \)
   (b) \( x = 1 \)

Continued...
7. (9 points) Let \( f(x) = \sqrt{2x - 1} \).

(a) Find the derivative \( f'(x) \).

\[
g(u) = \sqrt{u} \quad ; \quad g'(u) = \frac{1}{2\sqrt{u}}
\]

\[
u = h(x) = 2x - 1 \quad ; \quad h'(x) = 2
\]

\[
f'(x) = g'(u)h'(x) = \frac{1}{2\sqrt{u}} \cdot 2 = \frac{1}{\sqrt{u}} = \frac{1}{\sqrt{2x - 1}}
\]

(b) Find the equation of the tangent line to the graph \( y = f(x) \) at \( x = 1 \).

\[
y - y_0 = m(x - x_0)
\]

\[
x_0 = 1, \quad y_0 = f(x_0) = \sqrt{2 \cdot 1 - 1} = 1
\]

\[
m = f'(x_0) = \frac{1}{\sqrt{2 \cdot 1 - 1}} = 1
\]

\[
y - 1 = 1 \cdot (x - 1)
\]

\[
y - 1 = x - 1
\]

\[
y = x
\]

Continued...
(c) Sketch the tangent line from part (b) on the graph below. Find the \( y \)-intercept.

8. (bonus: 6 points) Christie runs out of flour for the cake she is baking so she drives to the nearest grocery store at the speed of 60 mph. When she arrives there she finds that the store is closed, so she drives back at the speed of 40 mph. What was Christie’s average speed for the trip?

\[
\text{Average speed} = \frac{\Delta s}{\Delta t} = \frac{\text{Distance traveled}}{\text{time}}
\]

Say the store is \( l \) miles from Christie’s home Then \( \Delta s = 2l \)

\[
\text{time} = \frac{l}{60} + \frac{l}{40}
\]

\[
= \frac{2l}{60 + 40} = \frac{2}{\frac{10}{6} + \frac{1}{4}} = \frac{20}{\frac{6}{4}} = \frac{20 \cdot 24}{10} = 2.24 \text{mph}
\]

The end.