Thu., Feb. 21, 2008 **MATH 255A** **Midterm Test 1**

Name: (print) Solutions

CSUN ID No. : ______

This test paper has 7 pages. The duration of the test is 1 hour 15 minutes. There are 6 questions to the total of 56 points.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	8	total

Note: Write solutions in the space provided. If you run out of space you may use the back side of the page. *Important: All answers should be justified. Show your work clearly and completely, explaining your steps.*

1. (6 points) Let $f(x) = 1 - x^2$ and $g(x) = \log_2 x$. Find

   (a) $f(g(x))$ and its domain

   $$f(g(x)) = 1 - (g(x))^2 = 1 - (\log_2 x)^2$$

   Domain: $x > 0$, or $(0, \infty)$.
   (for the log to be defined.)

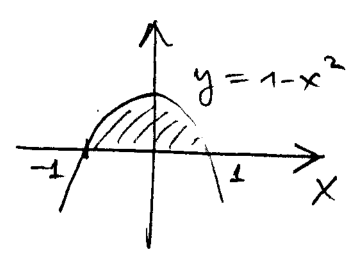
   (b) $g(f(x))$ and its domain

   $$g(f(x)) = \log_2(f(x)) = \log_2(1 - x^2)$$

   Domain. $1 - x^2 > 0$

   $-1 < x < 1$

   or $(-1, 1)$



2. (10 points) When an antibiotic is introduced into a culture of 50, 000 bacteria, the number of bacteria decreases exponentially. After 9 hours, there are only 20,000 bacteria

(a) Write an exponential equation to express the number of bacteria as a function of time in hours.

$$A(t) = A_0 e^{kt}$$

$A(9) = 20$ [thousand]

$A(0) = 50$ [thousand]

$$A_0 e^{k \cdot 0} = A_0 = 50$$

$$A_0 e^{k \cdot 9} = 50 e^{9k} = 20$$

$$e^{9k} = \frac{2}{5}$$

$$9k = \ln \frac{2}{5}$$

$$k = \frac{1}{9} \ln \frac{2}{5} \approx -0,1018$$

(b) In how many hours will half the number of bacteria remain?

Find t such that

$$A(t) = A_0 e^{kt} = 25$$

$$50 e^{kt} = 25$$

$$e^{kt} = \frac{1}{2}$$

$$kt = \ln \frac{1}{2}$$

$$t = \frac{1}{k} \ln \frac{1}{2} = 9 \frac{\ln \frac{1}{2}}{\ln \frac{2}{5}} \approx 6.8082$$

[6 hours 48 min]

3. (10 points) For the function

$$f(x) = \frac{x-1}{x+1}$$

(a) Find the domain, asymptotes and intercepts

$x+1=0 \iff x=-1$ – vertical asymptote.

$$\frac{x-1}{x+1} = \frac{1-\overset{\approx 0}{\frac{1}{x}}}{1+\underset{\approx 0}{\frac{1}{x}}} \approx 1 \text{ when } |x| \text{ is large}$$

when $|x|$ is large

$y=1$ – horizontal asymptote.

$x=0 \Rightarrow f(x)=-1 \Rightarrow y=-1$ (y-intercept)

$y=0 \Rightarrow x-1=0 \Rightarrow x=1$ (x-intercept)

(b) Find the intervals where $f(x) > 0$ and $f(x) < 0$

$\frac{x-1}{x+1} > 0 \Rightarrow$ either $x-1>0,\ x+1>0$

or $x-1<0,\ x+1<0$

$\Rightarrow$ $x>1$

or $x<-1$

$f(x)>0$ for $x>1$ or $x<-1$

$\left(x \in (-\infty,-1) \cup (1,\infty) \right)$

$\frac{x-1}{x+1} < 0 \Rightarrow$ either $x-1>0,\ x+1<0$

or $x-1<0,\ x+1>0$

$\Rightarrow$ $x>1,\ x<-1$ (impossible)

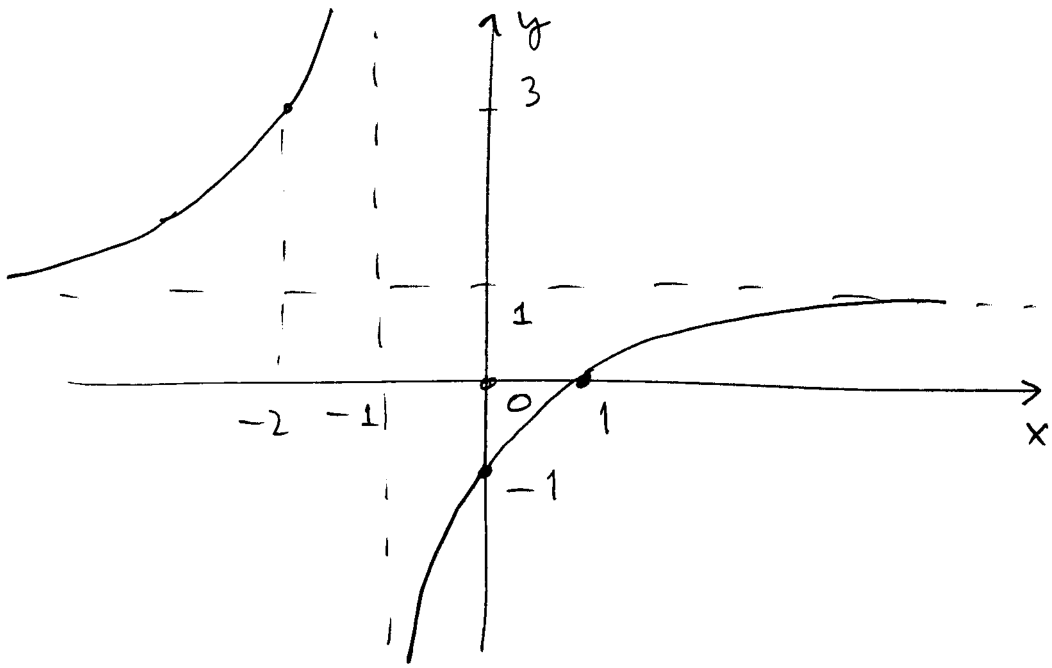
or $-1<x<+1$

$f(x)<0$ for $-1<x<1$

$\left(x \in (-1, 1) \right)$

Continued...

(c) Sketch the graph of the function



4. (12 points) Solve the equations

(a) $\frac{1}{27} = 3^{x-1}$

$$\frac{1}{3^3} = 3^{x-1}$$

$$3^{-3} = 3^{x-1}$$

$$-3 = x-1$$

$$x-1 = -3$$

$$x = -2$$

check: $\frac{1}{27} = 3^{-3}$ ✓

(b) $\ln x^2 + \ln x^4 + \ln x^6 = 0$

$$\ln x^{12} = 0$$

$$x^{12} = 1$$

$$x = 1 \quad \text{or} \quad x = -1 \quad \text{(both solutions)}$$

Check: $\ln 1 + \ln 1 + \ln 1 = 0$ ✓

(c) $e^{-0.5k} = 1 + e$

$$-0.5k = \ln(1+e)$$

$$k = -2\ln(1+e) \approx -2{,}6265$$

5. (12 points) The maximum afternoon temperature (in degrees F) in a given city is approximated by

$$T(x) = 65 + 28\cos\left(\frac{\pi x}{6}\right)$$

where x represents the month, $x = 0$ being January, $x = 1$ being February, and so on.

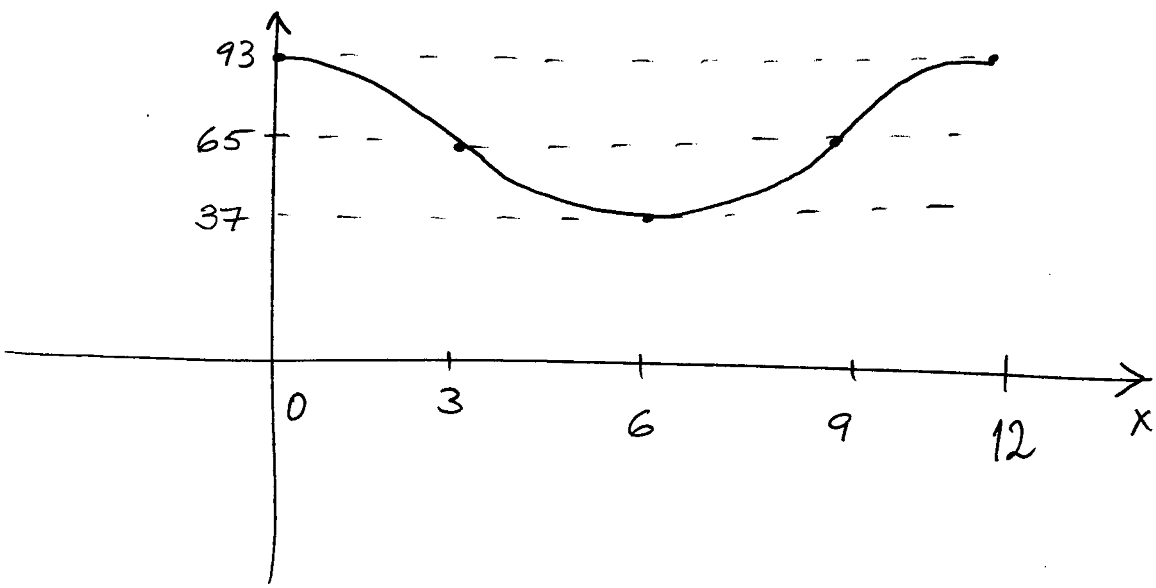
(a) Find the values of the temperature in March, June, and August.

JAN $\Rightarrow X = 2$ $\qquad T(2) = 65 + 28 \cos\frac{\pi}{3} = 79$

JUN $\Rightarrow X = 5$ $\qquad T(5) = 65 + 28 \cos\frac{5\pi}{6} \approx 40.75$

AUG $\Rightarrow X = 7$ $\qquad T(7) = 65 + 28 \cos\frac{7\pi}{6} \approx 40.75$

(b) Graph the function $T(x)$ over an interval of one period.



(c) Find the highest and the lowest yearly values of the temperature $T(x)$. In what months are they achieved?

Graph

max. temperature: $\cos(\ldots) = 1$

$\Rightarrow T = 65 + 28 = 93;$ $\left(X = 0 \text{ or } X = 12\right)$

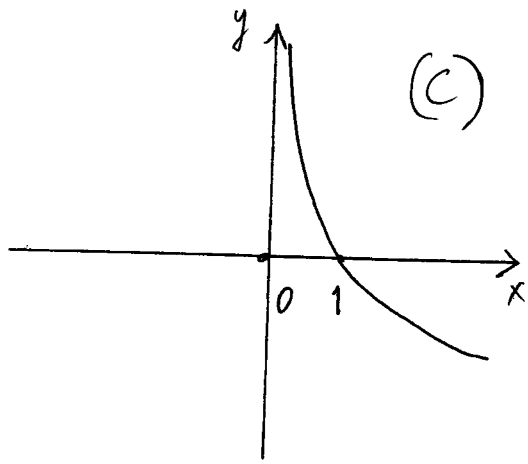
min. temperature $\cos(\ldots) = -1$

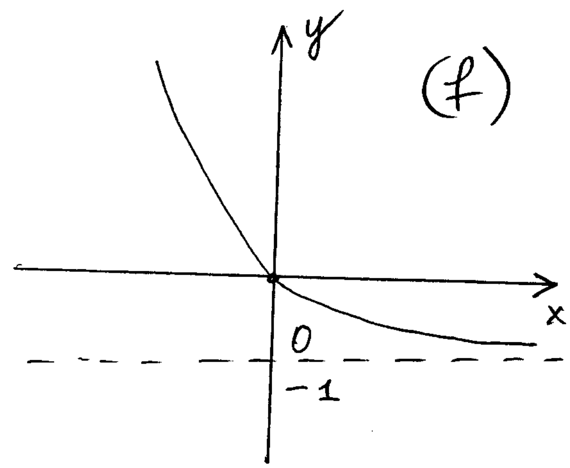
$\Rightarrow T = 65 - 28 = 37$ $(X = 6)$

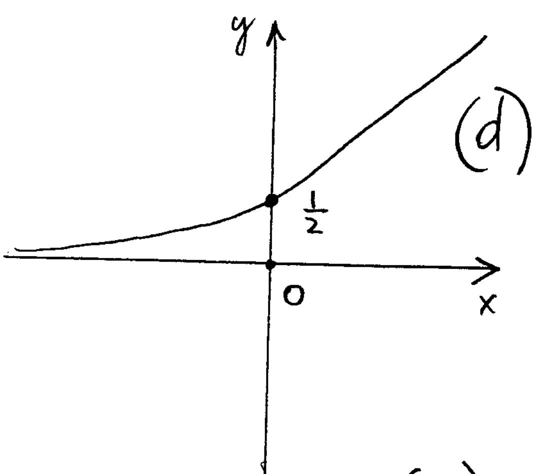
Continued...

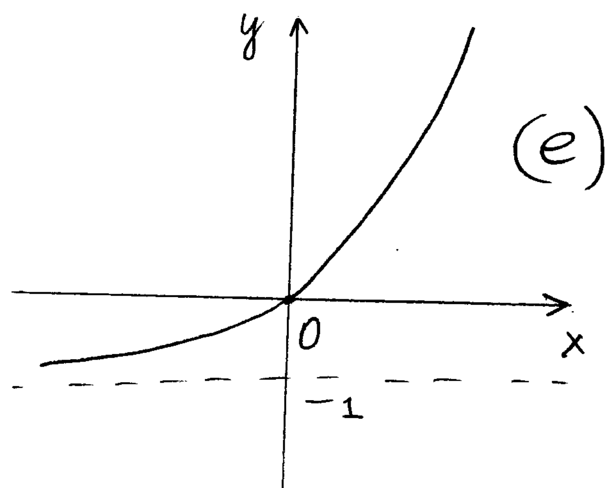
6. (6 points) In this problem you are given six functions and six graphs. Mark each graph with a letter (a), (b), and so on, to match them with the functions.

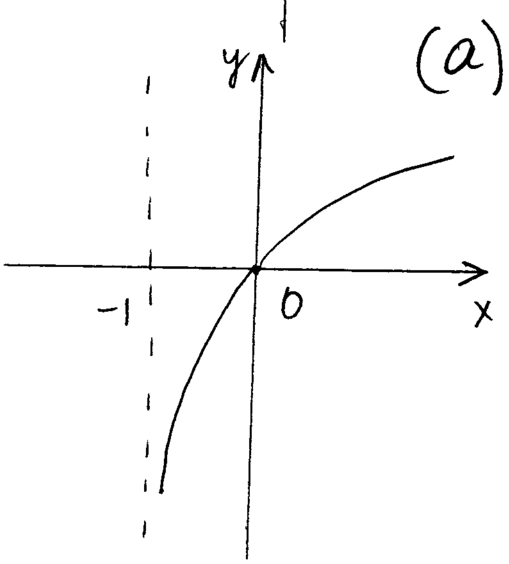
(a) $y = \log_2(x+1)$ (b) $y = \log_2(x-1)$ (c) $y = \log_2 \frac{1}{x}$
(d) $y = 2^{x-1}$ (e) $y = 2^x - 1$ (f) $y = 2^{-x} - 1$

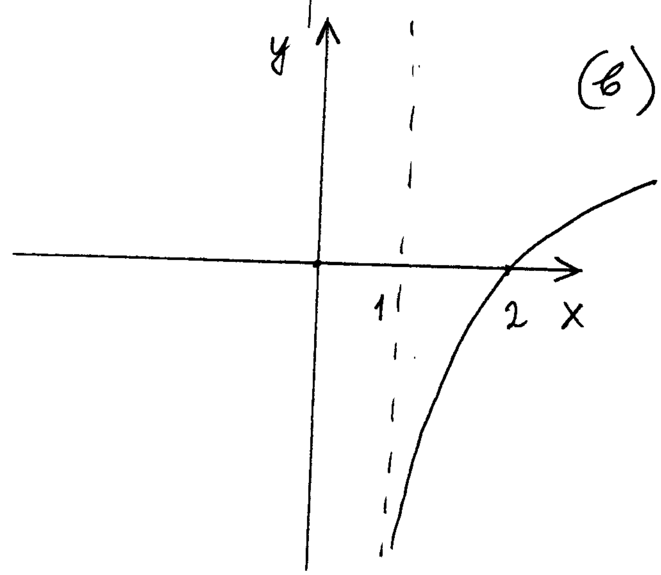












The end.