

Name: (print) Solutions

Justify your answers, show all work. Graphing calculators are not allowed. Closed books/notes.

1. (3 points) Using the definition of the derivative find $f'(x)$. Then find $f'(-2)$, $f'(0)$ and $f'(1)$ (if they are defined).

$$f(x) = -2/x.$$

Definition of the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-\frac{2}{x+h} - \left(-\frac{2}{x}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2(x+h) - 2x}{h \cdot x(x+h)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2}{x(x+h)} = \frac{2}{x^2}, \quad x \neq 0.$$

$$f'(-2) = \frac{2}{4} = \frac{1}{2}$$

$$f'(0) = \text{undefined}$$

$$f'(1) = \frac{2}{1} = 2$$

2. (2 points) Find the derivative of the function $y = \frac{5x+6}{\sqrt{x}}$ using an appropriate rule (sum/difference, power, or product/quotient, or their combination)

$$y = \frac{5x+6}{\sqrt{x}} = 5\sqrt{x} + \frac{6}{\sqrt{x}} = 5x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$$

$$y' = \frac{5}{2}x^{-\frac{1}{2}} - \frac{6}{2}x^{-\frac{3}{2}}$$

$$= \frac{5}{2\sqrt{x}} - 3\frac{1}{x\sqrt{x}}.$$

OR:

$$\frac{5x+6}{\sqrt{x}} = \frac{u}{v}$$

$$\left(\frac{5x+6}{\sqrt{x}}\right)' = \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} = \frac{5\sqrt{x} - (5x+6)\frac{1}{2\sqrt{x}}}{x}$$

$$u = 5x+6 ; u' = 5$$

$$v = \sqrt{x} ; v' = \frac{1}{2\sqrt{x}}$$

$$= \frac{\frac{5}{2}\sqrt{x} - \frac{3}{\sqrt{x}}}{x}$$

$$= \frac{5}{2\sqrt{x}} - \frac{3}{x\sqrt{x}}.$$

3. (2 points) If $g(3) = 4$, $g'(3) = 5$, $f(3) = 7$, and $f'(3) = 6$, find $h'(3)$ when $h(x) = f(x)g(x)$

Product rule:

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(3) = f'(3)g(3) + f(3)g'(3)$$

$$= 6 \cdot 4 + 7 \cdot 5$$

$$= 24 + 35 = \underline{\underline{59}}.$$