Fri., Nov. 16, 2007, **MATH 255A** **Midterm Test 3**

Name: (print) ______________________

Student ID No. : Solutions.

This test paper has 7 pages. The duration of the test is 50 minutes. There are 5 questions in the main part (46 points) and one bonus question (6 points).

Your scores: (do not enter answers here)

1	2	3	4	5	6	total

Note: Write solutions in the space provided. If you run out of space you may use the back of the page. *Important: All answers should be justified. Show your work clearly and completely, explaining your steps.*

1. (6 points) Find and classify all relative extrema of the function $g(x) = 2x - \ln x$.

$$g'(x) = 2 - \frac{1}{x}$$

$$g'(x) = 0 \Rightarrow 2 - \frac{1}{x} = 0 \Rightarrow 2 = \frac{1}{x} \Rightarrow x = \frac{1}{2}$$

(the only crit. point)

$$g''(x) = \frac{1}{x^2} > 0$$

by the 2nd derivative test $x = \frac{1}{2}$ is a point of local min.

(Since it is the only crit point it is also the global minimum)

2. (8 points) For the function $f(t) = \sin(\pi t)$

   (a) Find the third and the fourth order derivatives $f'''(t)$ and $f^{(4)}(t)$

$$f'(t) = \pi \cos(\pi t)$$
$$f''(t) = -\pi^2 \sin(\pi t)$$
$$f'''(t) = -\pi^3 \cos(\pi t)$$
$$f^{(4)}(t) = \pi^4 \sin(\pi t)$$

   (b) Find the values $f'''(\frac{1}{2})$ and $f^{(4)}(\frac{1}{2})$.

$$f'''\left(\frac{1}{2}\right) = -\pi^3 \cos\left(\frac{\pi}{2}\right) = 0$$
$$f^{(4)}\left(\frac{1}{2}\right) = \pi^4 \sin\left(\frac{\pi}{2}\right) = \pi^4$$

3. (12 points) For the function $f(x) = x^3 - 3x^2$

(a) Find the domain, and the x- and y-intercepts.

domain: all real x ($f(x)$ is a polynomial)

y-intercept: $x=0$, $y = f(0) = 0$

x-intercept: $y=0$, $x^3 - 3x^2 = 0$

$$x^2(x-3) = 0$$

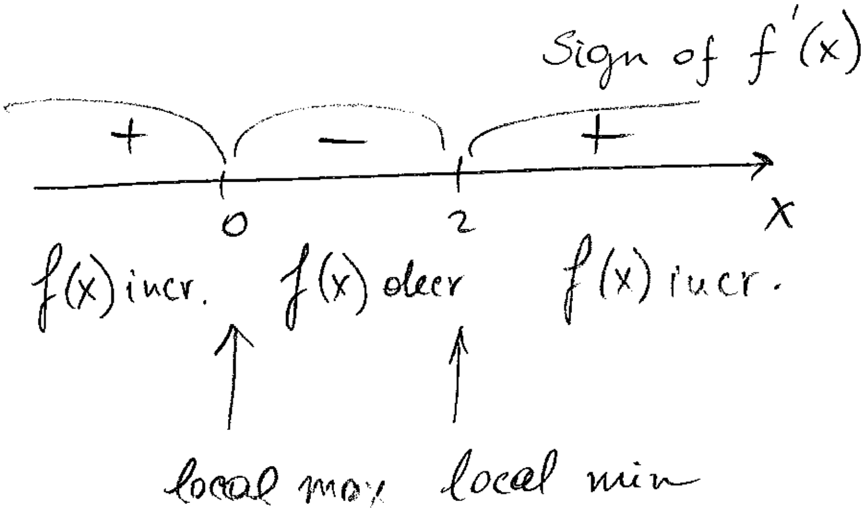
$$x = 0 \text{ or } x = 3$$

(b) Find the critical points and the intervals where $f(x)$ increases or decreases.

$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

$$f'(x) = 0 \Rightarrow 3x(x-2) = 0$$

$$x = 0 \text{ or } x = 2.$$



(c) Find the inflection points and the intervals of convexity/concavity.

$$f''(x) = 6x - 6 = 6(x-1)$$

$$f''(x) = 0 \Rightarrow x - 1 = 0$$

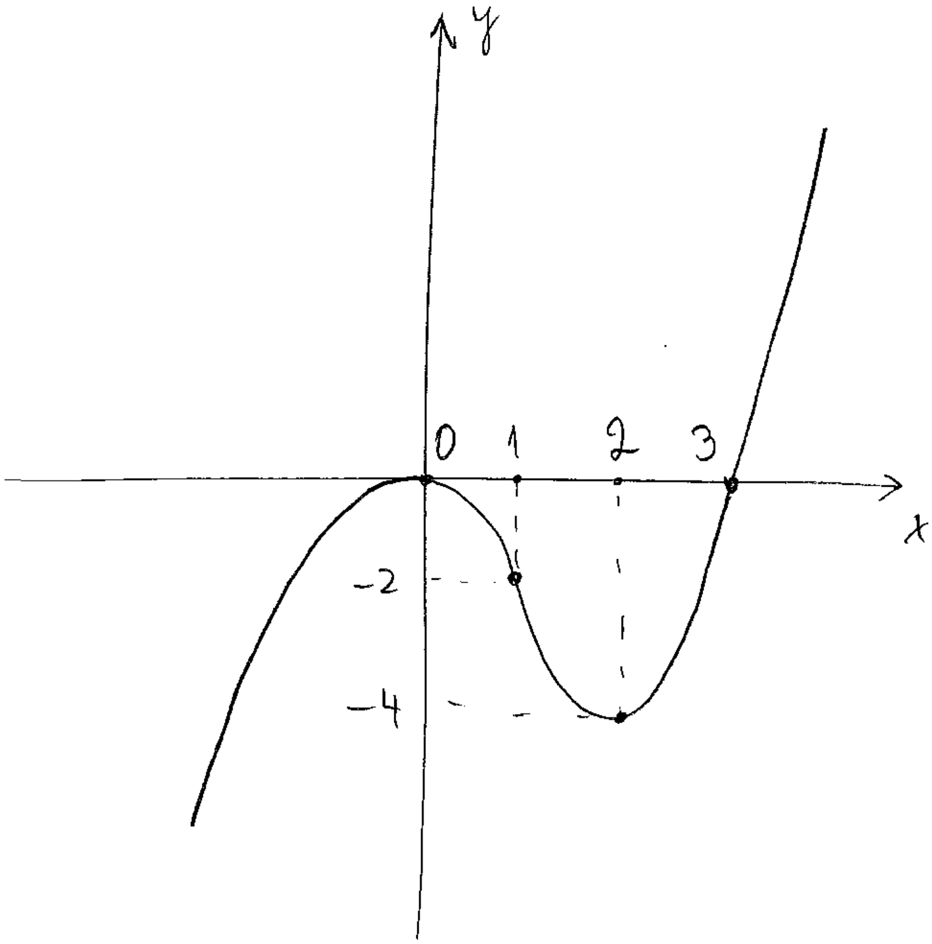
$$x = 1$$

$-$ 1 $+$

$f(x)$ concave $\cap$ $f(x)$ convex $\cup$

inflection pt.

(d) Using parts (a), (b) and (c) sketch the graph of $f(x)$. Hint: $f(0) = 0$, $f(1) = -2$, $f(3) = -4$, $f(3) = 0$.



Continued...

4. (10 points) The concentration of a drug in patient's bloodstream x hours after the drug is administered is given by

$$K(x) = \frac{2x}{x^2+1} \qquad [mg/l].$$

(a) When will the concentration start to decrease?

$$K'(x) = 2\,\frac{x^2+1-2x^2}{(x^2+1)^2} = 2\,\frac{1-x^2}{(x+1)^2}$$

$$K'(x) = 0 \Rightarrow \quad 1-x^2=0$$

$$x^2 = 1$$

$$x = \pm 1$$

(the "meaningful domain" is $x > 0$, so $x = 1$)

$$\underset{0}{|} \quad + \quad \underset{1}{|} \quad - \quad \longrightarrow \qquad \text{Sign of } K'(x)$$

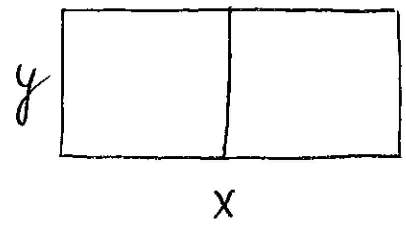
$K(x)$ decreases after $x = 1$ (hours)

(b) Find the maximum concentration.

$x = 1$ is the global max

$$K(1) = \frac{2}{1+1} = 1 \ [mg/l].$$

5. (10 points) A rectangular fence with one middle partition (see figure) is to be built using 1200 ft of fencing. Find the dimensions of the fence that has the maximum area.



$S = xy$ — area

$L = 3y + 2x$ — total length.

maximize $S = xy$

subject to $L = 3y + 2x = 1200$.

Express y in terms of x:

$$3y + 2x = 1200$$

$$y = \frac{1200 - 2x}{3}$$

$$S(x) = x\,\frac{1200 - 2x}{3} = \frac{1}{3}\left(1200x - 2x^2\right)$$

$$S'(x) = \frac{1}{3}\left(1200 - 4x\right) = 0$$

$$1200 - 4x = 0$$

$$4x = 1200$$

$$x = 300 \text{ [ft]}$$

$S''(x) = -\frac{4}{3} < 0 \Rightarrow$ $x = 300$ is local max.

(Since $x = 300$ is the only crit. pt. it is a point of global max.)

Continued...

$$y = \frac{1200 - 2 \cdot 300}{3} = 200 \text{ [ft]}.$$

Answer: 300 ft by 200 ft.

6. (bonus problem: 6 points) The function

$$f(x) = (\tan x - \sin x)^2$$

has an extremum point at $x = 0$. Find out if this is a maximum or a minimum.

$f(x) \geq 0$ ($f(x)$ is a square)

$f(0) = (\tan 0 - \sin 0)^2 = (0-0)^2 = 0.$

$\Rightarrow$ $x=0$ is a minimum.