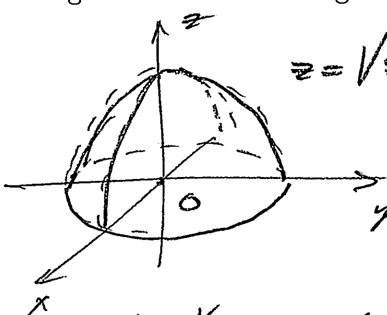
Mon. Nov 9, 2015 **MATH 250, Quiz 8** Prof. V. Panferov

Name (print): *Solutions.*

Each problem is worth 2 points. Show all your work.

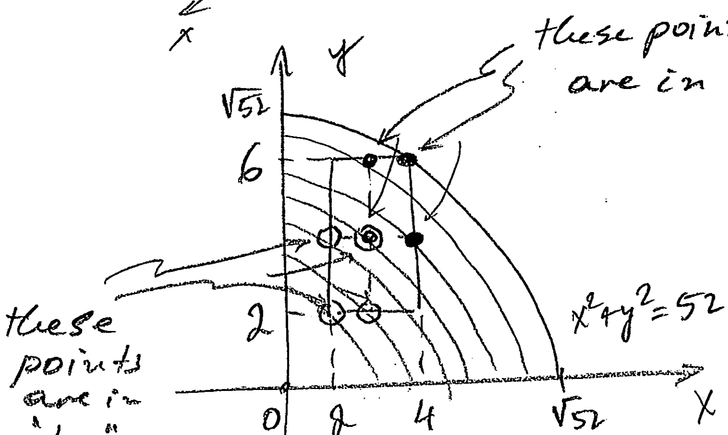
1. Let V be the volume of the solid that lies under the graph of $f(x, y) = \sqrt{52 - x^2 - y^2}$ and above the rectangle $R = [2, 4] \times [2, 6]$. We use the lines $x = 3$ and $y = 4$ to divide R into subrectangles. Let L and U be the Riemann sums computed using lower left corners and upper right corners, respectively. Without calculating the numbers V, L and U, arrange them in increasing order and explain your reasoning.



$z = \sqrt{52 - x^2 - y^2}$ (upper half of a sphere.)

The function $f(x,y)$ decreases as x^2+y^2 increases.

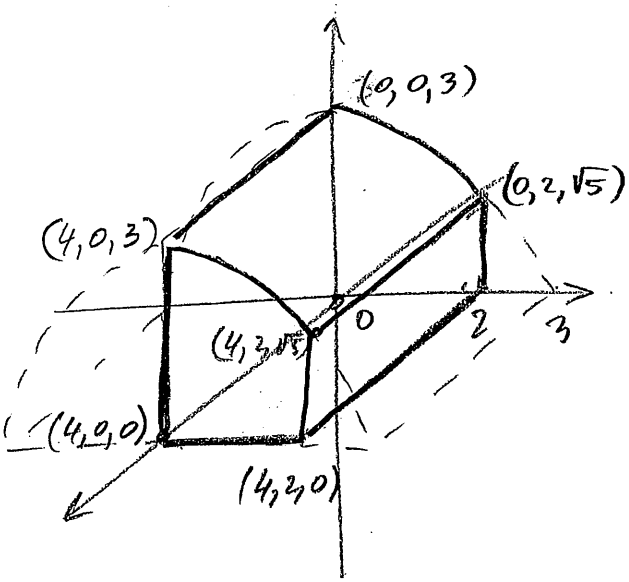
For each subrectangle the values are between the max at lower left corner and min at upper right corner



$\Rightarrow$ L is an "over-estimate" of the volume
U is an "underestimate"

where $R = [0,2] \times [0,4]$ $\Rightarrow U < V < L$.

2. The integral $\iint_R \sqrt{9 - y^2}\, dA$ represents the volume of a solid. Sketch the solid.



$z = \sqrt{9-y^2}$

$z^2 = 9 - y^2 \quad (z \geq 0)$

$y^2 + z^2 = 9 \quad (z \geq 0)$

upper half of the circle in the yz-plane.

$z = f(x,y)$ does not depend on $x \Rightarrow$ graph is a cylinder

Please turn over...

3. Find the volume of the solid lying under the elliptic paraboloid $x^2/4 + y^2/9 + z = 1$ and above the rectangle $R = [-1, 1] \times [-2, 2]$.

$$V = \int_{-1}^{1}\int_{-2}^{2} \left(1 - \frac{x^2}{4} - \frac{y^2}{9}\right) dy\,dx$$

$$= \int_{-1}^{1} 4\cdot\left(1-\frac{x^2}{4}\right) - \frac{2^3+2^3}{27}\,dx$$

$$= \int_{-1}^{1} 4 - x^2 - \frac{16}{27}\,dx$$

$$= 2\cdot\left(4 - \frac{16}{27}\right) - \int_{-1}^{1} x^2\,dx$$

$$= 2\cdot\frac{108-16}{27} - \frac{1^3+1^3}{3}$$

$$= 2\cdot\frac{108-16-9}{27} = \frac{166}{27}$$